# Electric Circuits

# & Diff Eq's of 2nd order

- Faraday's Law


L: inductance
i: electric current

$$\text{(voltage drop across the inductor)} = L\frac{di}{dt}$$

- Ohm's Law


R: resistance
i: electric current

$$\text{(voltage drop across the resistor)} = Ri$$

- Coulomb's Law

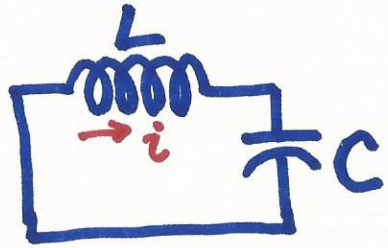

C: capacitance
Q: electric charge on capacitor

- $\text{(voltage drop across the capacitor)} = \frac{Q}{C}$
- electric current $i = \frac{dQ}{dt}$

- Kirchhoff's Voltage Law

$$\text{(the sum of voltage drops in any loop)} = 0$$

# LC Circuit




L : inductor
C : capacitor
$i$ : electric current
Q : electric charge on the capacitor
V : voltage drop across the capacitor

$$(*) \quad L\frac{di}{dt} + \frac{Q}{C} = 0$$

$$\begin{cases} i = \dfrac{dQ}{dt} \\ V = \dfrac{Q}{C} \end{cases} \Rightarrow i = C\frac{dV}{dt}$$

## A 2nd order Diff Eq for Q

$$L\frac{d^2Q}{dt^2} + \frac{1}{C}Q = 0$$

## A 2nd order Diff Eq for $i$

$$L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

## A 2nd order Diff Eq for V

$$LC\frac{d^2V}{dt^2} + V = 0$$

# LC Circuit


(*) $L\frac{di}{dt} + \frac{Q}{C} = 0$

$L$ : inductor

$C$ : capacitor

$i$ : electric current

$Q$ : electric charge on the capacitor

$v$ : voltage drop across the capacitor

$$\begin{cases} i = \frac{dQ}{dt} \\ v = \frac{Q}{C} \end{cases} \Rightarrow i = C\frac{dv}{dt}$$

## A System of Diff Eq's for Q and i

$$\begin{cases} \frac{dQ}{dt} = i \\ \frac{di}{dt} = -\frac{1}{LC} Q \end{cases}$$

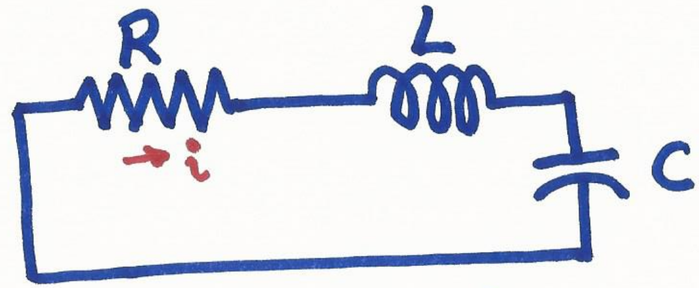
$$\frac{d}{dt}\begin{bmatrix} Q \\ i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & 0 \end{bmatrix}\begin{bmatrix} Q \\ i \end{bmatrix}$$

## A System of Diff Eq's for v and i

$$\begin{cases} \frac{dv}{dt} = \frac{1}{C} i \\ \frac{di}{dt} = -\frac{1}{L} v \end{cases}$$

$$\frac{d}{dt}\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & 0 \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix}$$

# RLC circuit




$$\begin{cases} i = \frac{dQ}{dt} \\ v = \frac{Q}{C} \end{cases} \Rightarrow i = C\frac{dv}{dt}$$

$$(*) \quad Ri + L\frac{di}{dt} + \frac{Q}{C} = 0$$

**Diff Eq for Q** (electric charge on the capacitor)

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{1}{C}Q = 0$$

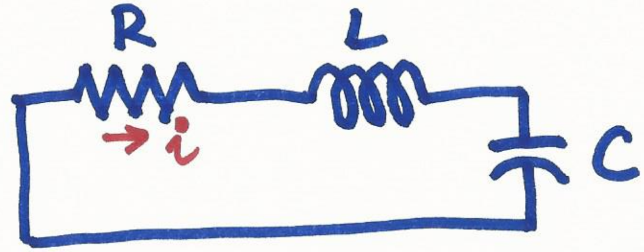
**Diff Eq for i** (electric current)

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = 0$$

**Diff Eq for v** (voltage drop across the capacitor)

$$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v = 0$$

# RLC Circuit




$$(*)\quad Ri + L\frac{di}{dt} + \frac{Q}{C} = 0$$

$$\begin{cases} i = \dfrac{dQ}{dt} \\ V = \dfrac{Q}{C} \end{cases} \Rightarrow i = C\frac{dv}{dt}$$

## A System of Diff Eq's for $\begin{cases} Q \text{ (electric charge on the capacitor)} \\ i \text{ (electric current)} \end{cases}$

$$\begin{cases} \dfrac{dQ}{dt} = i \\ \dfrac{di}{dt} = -\dfrac{1}{LC}Q - \dfrac{R}{L}i \end{cases}$$

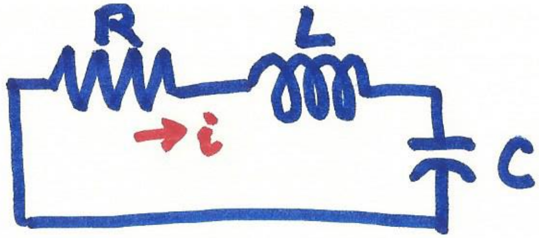
$$\frac{d}{dt}\begin{bmatrix} Q \\ i \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} Q \\ i \end{bmatrix}$$

## A System of Diff Eq's for $\begin{cases} v \text{ (voltage drop across the capacitor)} \\ i \text{ (electric current)} \end{cases}$

$$\begin{cases} \dfrac{dv}{dt} = \dfrac{1}{C}i \\ \dfrac{di}{dt} = -\dfrac{1}{L}v - \dfrac{R}{L}i \end{cases}$$

$$\frac{d}{dt}\begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}\begin{bmatrix} v \\ i \end{bmatrix}$$

# RLC Circuit




- 2nd Order Diff Eq for $i$ (electric current): $L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{1}{C} i = 0$
- Charecteristic Polynomial and Eigenvalues:

$$L\lambda^2 + R\lambda + \frac{1}{C} = 0. \qquad \lambda = \frac{-R \pm \sqrt{R^2 - 4L/C}}{2L}$$

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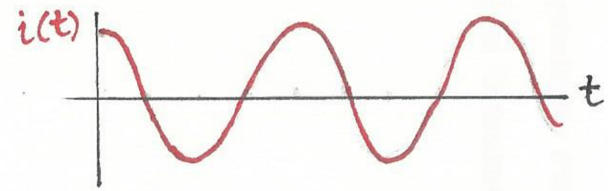
- Un-Damped Circuit

$R=0$ i.e. LC circuit

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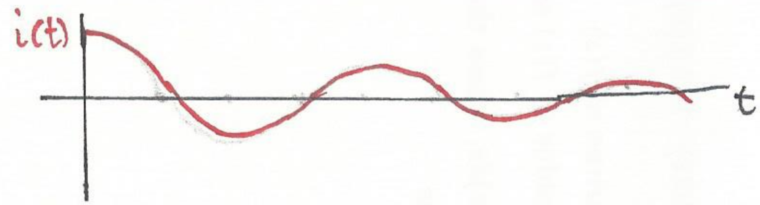
- Under-Damped Circuit

$R>0, \ R^2 - 4\frac{L}{C} < 0$

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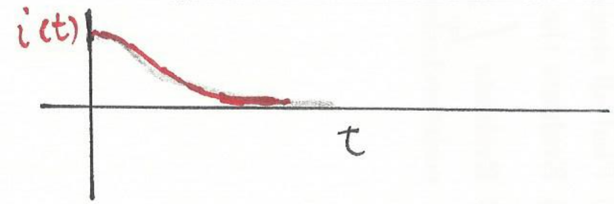
- Critically-Damped Circuit

$R^2 - 4\frac{L}{C} = 0.$ i.e. $R = 2\sqrt{\frac{L}{C}}$

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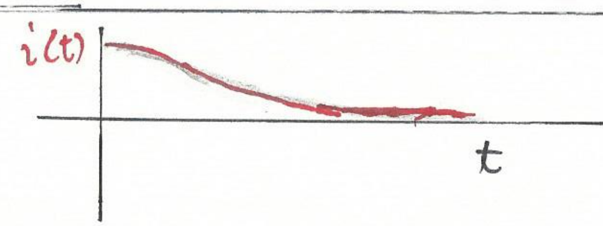
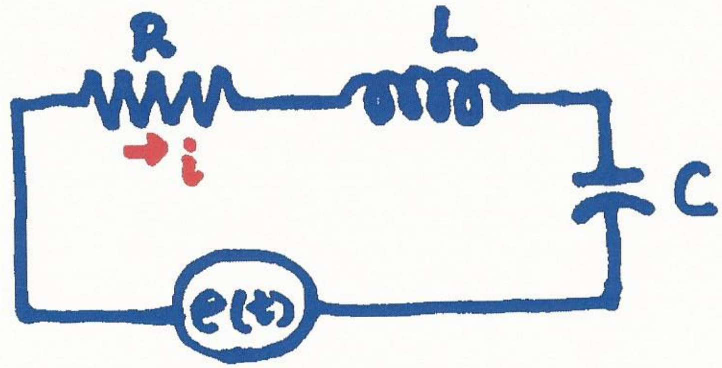
- Over-Damped Circuit

$R>0, \ R^2 - 4\frac{L}{C} > 0$



# RLC Circuit

with Voltage Source




Voltage Source

$$(*) \quad Ri + L\frac{di}{dt} + \frac{Q}{C} - e(t) = 0$$

$i = \frac{dQ}{dt}$

$v = \frac{Q}{C}$

**Diff Eq for Q** (electric charge on the capacitor)

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = e(t)$$

Nonhomogeneous Linear Diff Eq

**Diff Eq for i** (current)

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{1}{C}i = e'(t)$$

**Diff Eq for v** (voltage drop across the capacitor)

$$LC\frac{d^2v}{dt^2} + RC\frac{dv}{dt} + v = e(t)$$