Variation of Parameters. Reduction of Order.

Consider

$$x^2y'' + 2xy' - 2y = x.$$

1. Verify that $y_1(x) = x$ is a particular solution of the corresponding homogeneous equation

$$x^2y'' + 2xy' - 2y = 0.$$

2. Find all solutions of the homogeneous equation.

3. Find all solutions of the nonhomogeneous equation.

ANSWERS:

1. DIY (just plug the function in the homogeneous equation to see whether the equation is satisfied or not)

2. $C_1x + C_2x^{-2}$ where C_1, C_2 are arbitrary constants

3. $\frac{1}{3}x\ln|x| - \frac{1}{9}x + C_1x + C_2x^{-2}$ where C_1, C_2 are arbitrary constants