**Phase Portraits of 2-D Linear Systems with Zero Eigenvalue**

For each of the following systems,

- Find general solutions;
- skecth the phase portrait;
- determine whether the equilibrium $(x, y) = (0, 0)$ is stable or unstable;
- determine whether the equilibrium $(x, y) = (0, 0)$ is asymptotically stable.

[1] $x' = x - 2y, \quad y' = 3x - 6y.$

[2] $x' = -x + 2y, \quad y' = -3x + 6y.$

[3] $x' = -2x - 4y, \quad y' = x + 2y.$

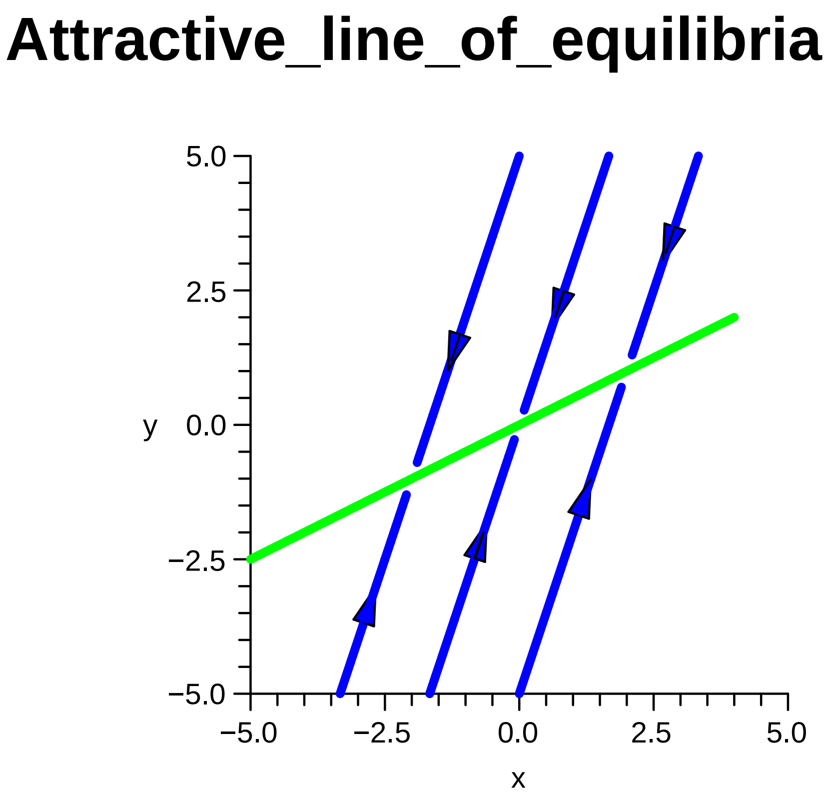
Try by yourself before looking at the answers

**Answers**

[1] General solutions: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 e^{-5t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

$(x, y) = (0, 0)$ is stable but is not asymptotically stable.

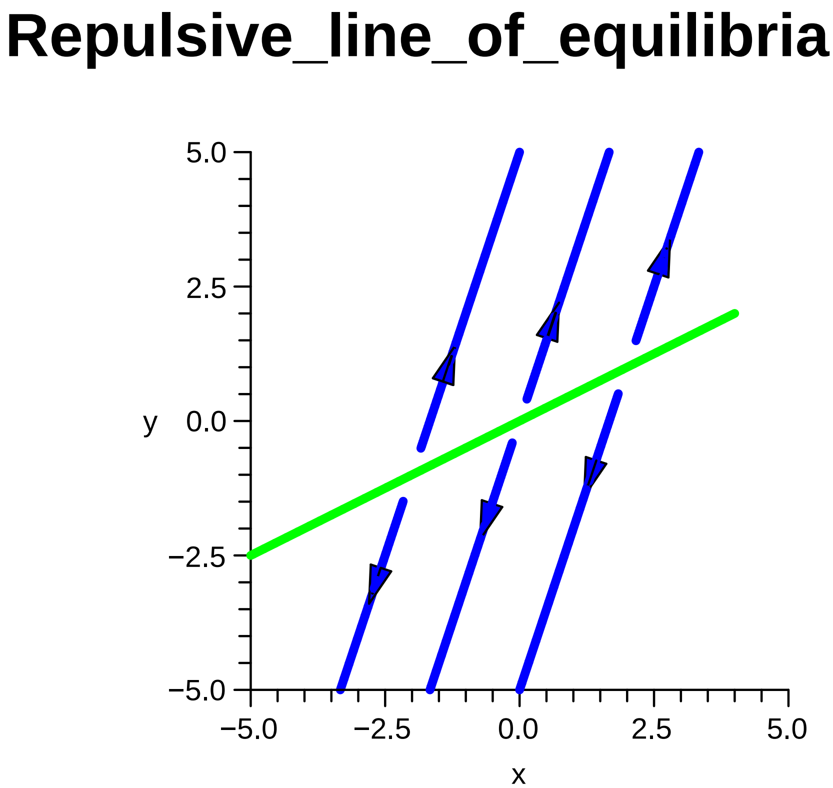
In the phase portrait below, every point on the green line is an equilibrium solution.



[2] General solutions: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + C_2 e^{5t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

$(x, y) = (0, 0)$ is unstable and (hence automatically) is not asymptotically stable.

In the phase portrait below, every point on the green line is an equilibrium solution.



[3] General solutions: $\begin{bmatrix} x \\ y \end{bmatrix} = C_1 \begin{bmatrix} -2 \\ 1 \end{bmatrix} + C_2 \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right)$.

$(x, y) = (0, 0)$ is unstable and is not asymptotically stable.

In the phase portrait below, every point on the green line is an equilibrium solution.

**Laminated_flow**