Some Applications of 2-D Systems of Differential Equations

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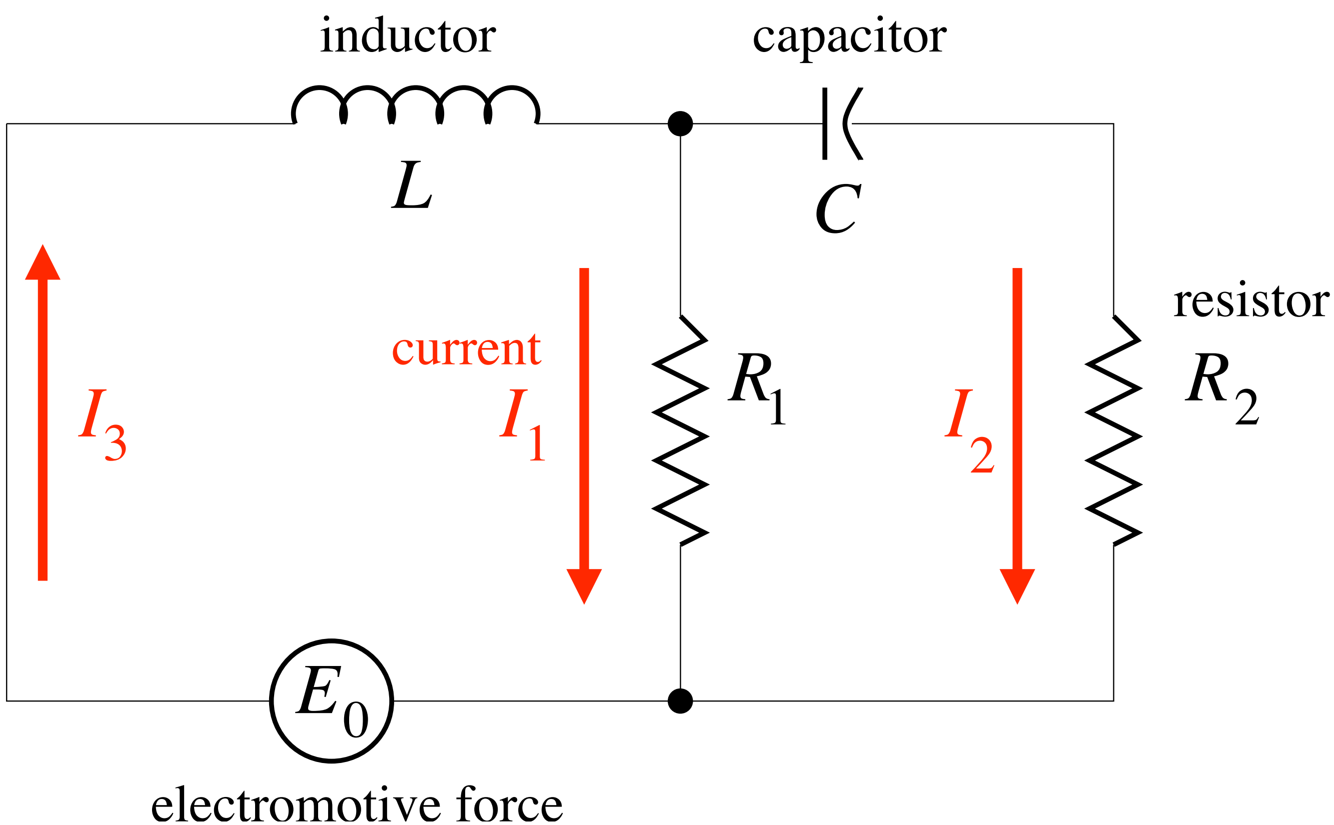
Examples:

- Salt in Tanks (a linear system)
- Electric Circuits (a linear system)
- Population Model - Competing Species (a nonlinear system)

Example 2. (Electric Circuit)

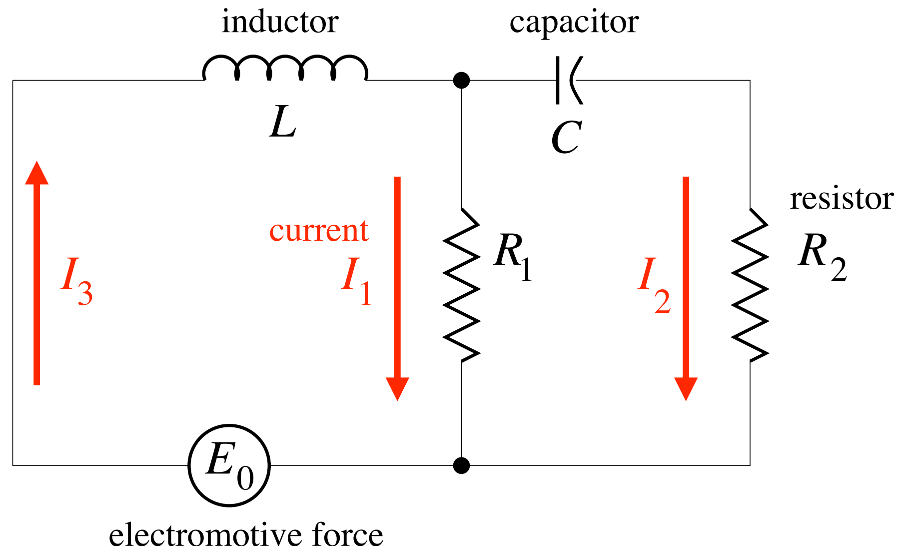


Question: Given $\begin{cases} R_1 = 50 \text{ ohms}, & R_2 = 25 \text{ ohms}, \\ L = 2 \text{ henries}, & C = 0.008 \text{ farads}, \quad E_0 = 100 \text{ volts}, \end{cases}$

and the initial condition $I_1(0) = I_2(0) = I_3(0) = 0$ amps,

find electric currents $I_1(t), I_2(t), I_3(t)$.

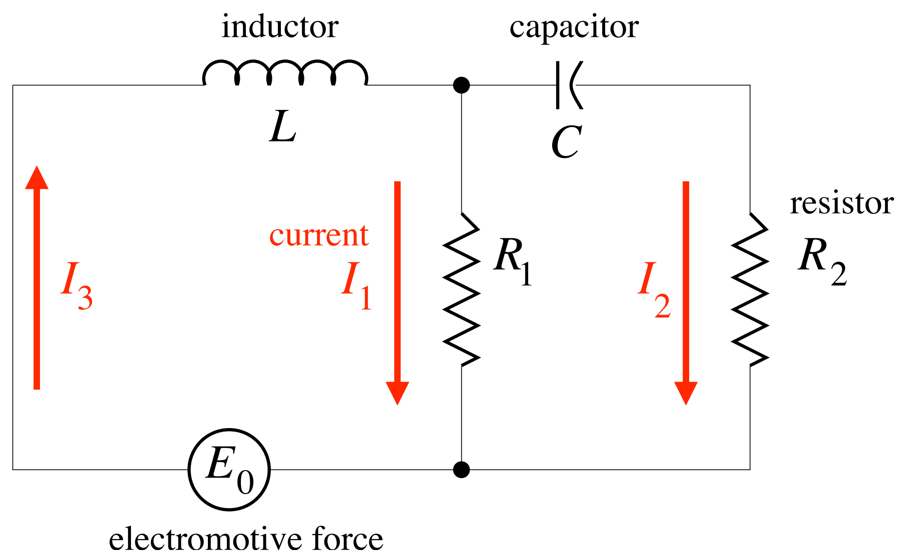
Example 2. (continued. Review of Physics.)



Definition of Current:

$$I = \frac{dQ}{dt} \quad \left(\begin{array}{c} \text{the rate of change} \\ \text{of charge } Q \end{array} \right)$$

Example 2. (continued. Review of Physics.)



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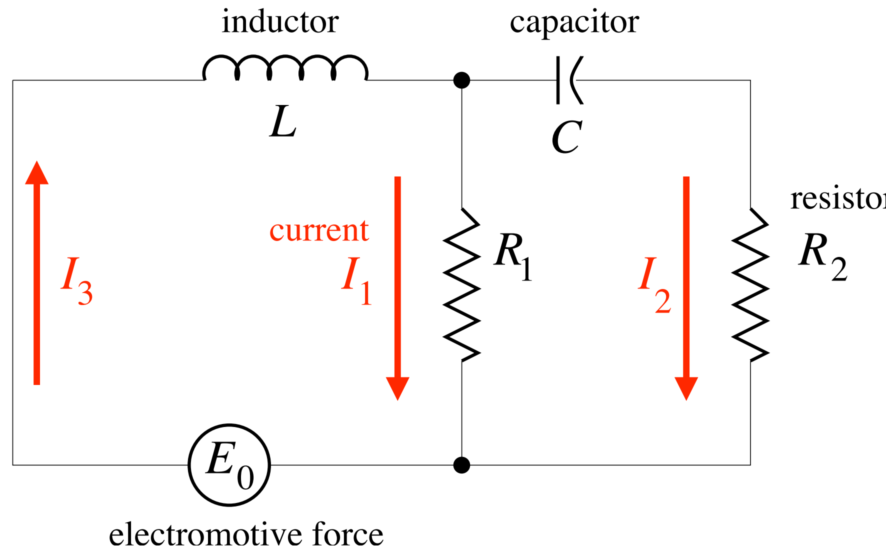
Circuit Element	inductor	resistor	capacitor
Voltage Drop	$L\,dI/dt$ (Faraday's law)	$R\,I$ (Ohm's law)	Q/C (Coulomb's law)

Example 2. (continued. Review of Physics.)



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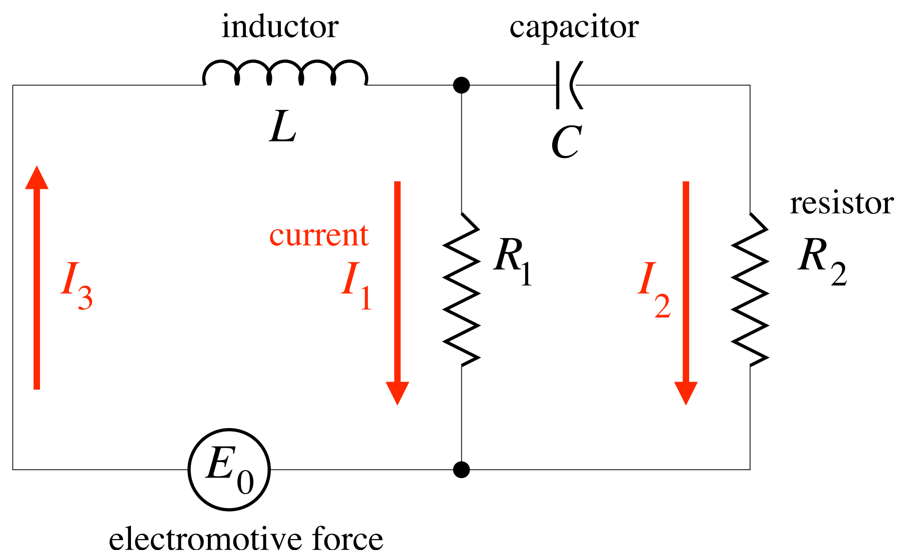
Kirchhoff's Current Law: $I_1 + I_2 = I_3$

Example 2. (continued. Review of Physics.)



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Kirchhoff's Voltage Law:

(the sum of voltage drops in a closed loop) $= 0$

Example 2. (continued. Apply Kirchhoff's law.)

$$I = dQ/dt$$

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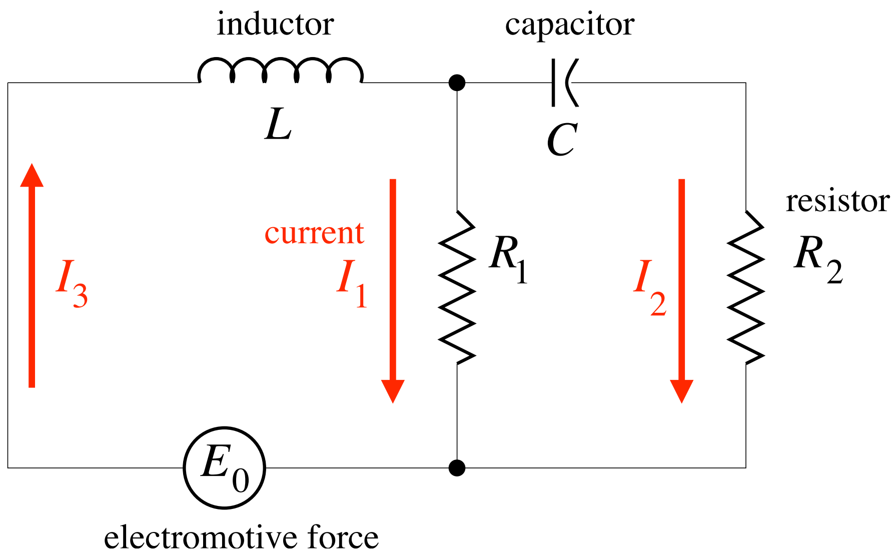

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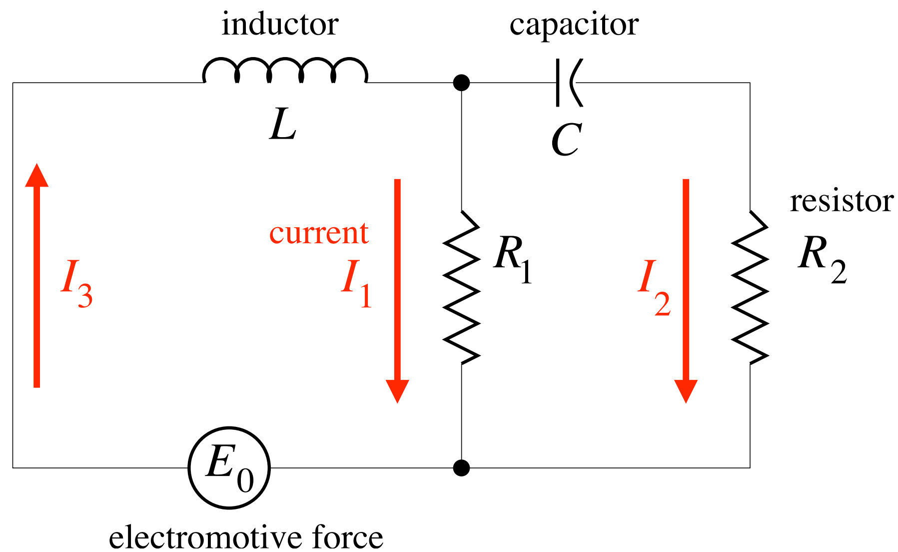

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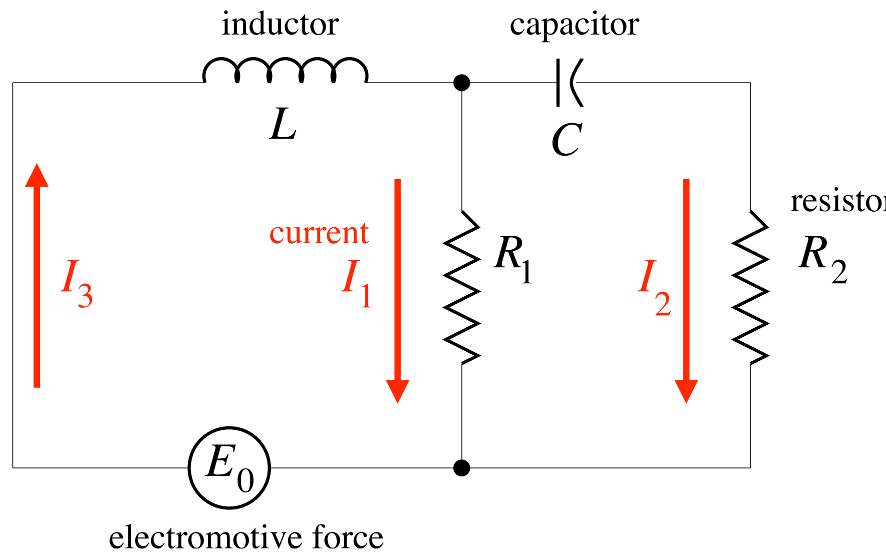
Voltage Drop	$L\,dI/dt$	$R\,I$	Q/C

Kirchhoff's Voltage Law
(the sum of voltage drops in a closed loop) $= 0$

The Left Loop

$$L\frac{dI_3}{dt} + R_1 I_1 - E_0 = 0$$

Example 2. (continued. Apply Kirchhoff's law.)



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(the sum of voltage drops in a closed loop) = 0

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$$L\frac{dI_3}{dt} + R_1 I_1 - E_0 = 0$$

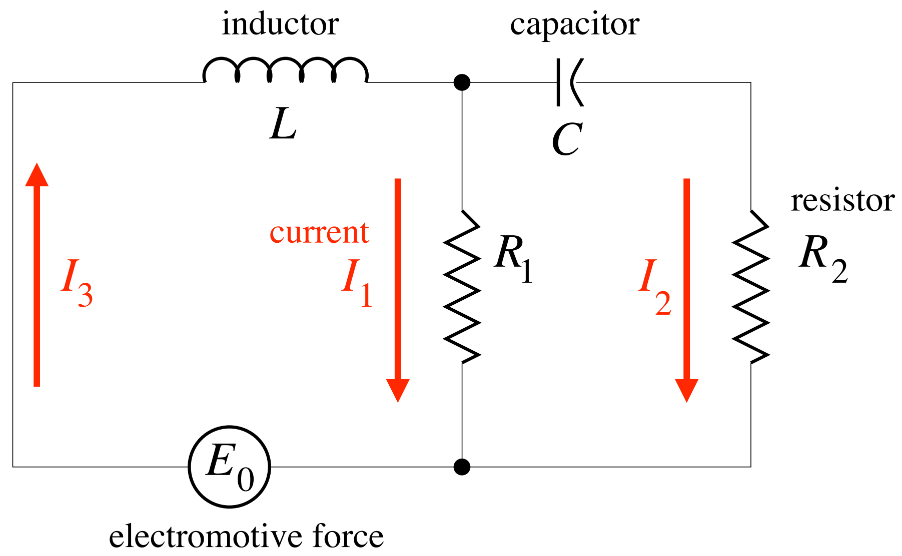
The Right Loop

$$\frac{1}{C}Q_2 + R_2 I_2 - R_1 I_1 = 0$$

Example 2. (continued. Set up equations.)

$$I = dQ/dt$$

$$I_1 + I_2 = I_3$$



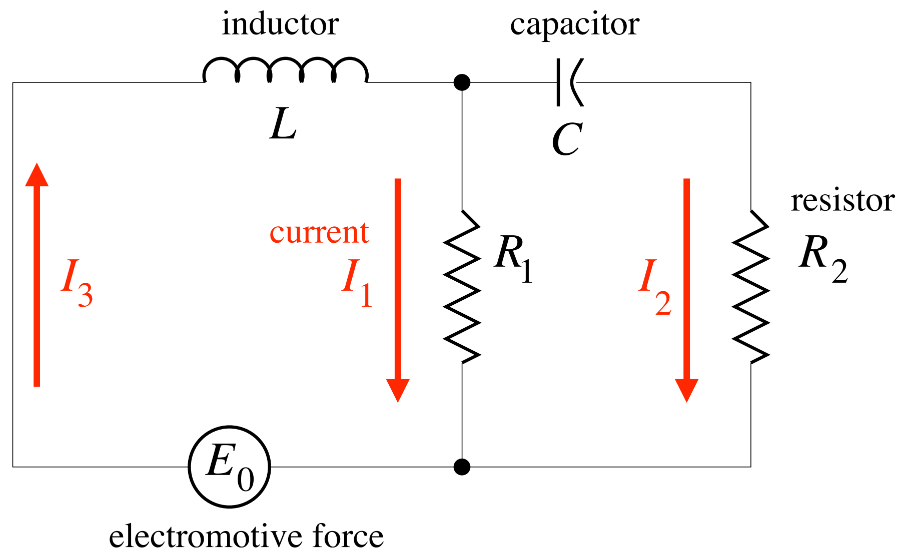
The Left Loop

$$L\frac{dI_3}{dt} + R_1 I_1 - E_0 = 0$$

The Right Loop

$$\frac{1}{C}Q_2 - R_1 I_1 + R_2 I_2 = 0$$

Example 2. (continued. Set up equations.)



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$$I_1 + I_2 = I_3$$

The Left Loop

$$L\frac{dI_3}{dt} + R_1 I_1 - E_0 = 0$$

$$\Downarrow \quad I_3 = I_1 + I_2$$

$$L\frac{dI_1}{dt} + L\frac{dI_2}{dt} + R_1 I_1 - E_0 = 0$$

The Right Loop

$$\frac{1}{C}Q_2 - R_1 I_1 + R_2 I_2 = 0$$

Example 2. (continued. Set up equations.)

$$I = dQ/dt$$

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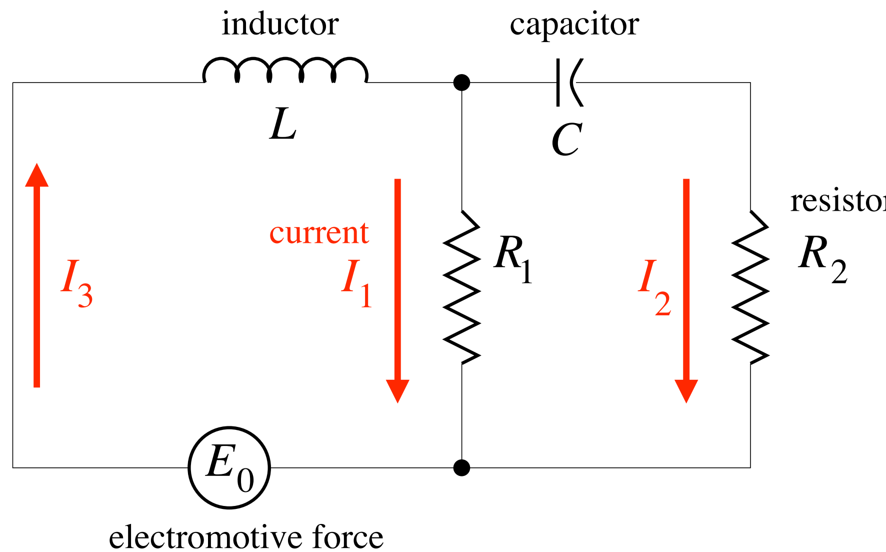

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$$L\frac{dI_3}{dt} + R_1 I_1 - E_0 = 0$$

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The Right Loop

$$\frac{1}{C}Q_2 - R_1 I_1 + R_2 I_2 = 0$$

$$\Downarrow \quad \frac{d}{dt}$$

$$\frac{1}{C}I_2 - R_1\frac{dI_1}{dt} + R_2\frac{dI_2}{dt} = 0$$

Example 2. (continued. Set up equations.)

$$\begin{cases} LI_1' + LI_2' + R_1 I_1 - E_0 = 0 \\ \frac{1}{C} I_2 - R_1 I_1' + R_2 I_2' = 0 \end{cases}$$

Example 2. (continued. Set up equations.)

$$\begin{cases} LI_1' + LI_2' + R_1 I_1 - E_0 = 0 \\ \frac{1}{C} I_2 - R_1 I_1' + R_2 I_2' = 0 \end{cases} \qquad \begin{cases} LI_1' + LI_2' = -R_1 I_1 + E_0 \\ -R_1 I_1' + R_2 I_2' = -\frac{1}{C} I_2 \end{cases}$$

Example 2. (continued. Set up equations.)

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Equilibrium: $\begin{cases} 0 = -R_1 I_1 + E_0 \\ 0 = -\frac{1}{C} I_2 \end{cases} \Rightarrow \begin{cases} I_1 = E_0/R_1 \\ I_2 = 0 \end{cases}$

Example 2. (continued. Set up equations.)

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Time-Dependent Equations in Matrix Form:

$$\begin{bmatrix} L & L \\ -R_1 & R_2 \end{bmatrix} \begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} -R_1 & 0 \\ 0 & -\frac{1}{C} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} E_0 \\ 0 \end{bmatrix}$$

Example 2. (continued. Set up equations.)

$$\begin{cases} LI_1' + LI_2' + R_1 I_1 - E_0 = 0 \\ \frac{1}{C} I_2 - R_1 I_1' + R_2 I_2' = 0 \end{cases} \qquad \begin{cases} LI_1' + LI_2' = -R_1 I_1 + E_0 \\ -R_1 I_1' + R_2 I_2' = -\frac{1}{C} I_2 \end{cases}$$

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Example 2. (continued. Set up equations.)

$$\begin{cases} LI_1' + LI_2' + R_1 I_1 - E_0 = 0 \\ \frac{1}{C} I_2 - R_1 I_1' + R_2 I_2' = 0 \end{cases} \qquad \begin{cases} LI_1' + LI_2' = -R_1 I_1 + E_0 \\ -R_1 I_1' + R_2 I_2' = -\frac{1}{C} I_2 \end{cases}$$

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2-D System: $\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} -\frac{R_1 R_2}{L(R_1+R_2)} & \frac{1}{C(R_1+R_2)} \\ -\frac{R_1^2}{L(R_1+R_2)} & \frac{1}{C(R_1+R_2)} \end{bmatrix} \begin{bmatrix} I_1 - \frac{E_0}{R_1} \\ I_2 \end{bmatrix}$

Example 2. (continued. Plug in numbers.)

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Plug in $\begin{cases} R_1 = 50 \text{ ohms}, & R_2 = 25 \text{ ohms}, \\ L = 2 \text{ henries}, & C = 0.008 \text{ farads}, \quad E_0 = 100 \text{ volts}, \end{cases}$

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2-D System: $$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} -25/3 & 5/3 \\ -50/3 & -5/3 \end{bmatrix} \begin{bmatrix} I_1 - 2 \\ I_2 \end{bmatrix}$$

Example 2. (continued. Solve the system.)

Initial Value Problem

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} -25/3 & 5/3 \\ -50/3 & -5/3 \end{bmatrix} \begin{bmatrix} I_1 - 2 \\ I_2 \end{bmatrix}, \qquad \begin{bmatrix} I_1(0) \\ I_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Example 2. (continued. Solve the system.)

Initial Value Problem

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} -25/3 & 5/3 \\ -50/3 & -5/3 \end{bmatrix} \begin{bmatrix} I_1 - 2 \\ I_2 \end{bmatrix}, \qquad \begin{bmatrix} I_1(0) \\ I_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

Eigenvalues: $\lambda_1 = -5 + \frac{5}{3}\sqrt{6}\,i, \quad \lambda_2 = -5 - \frac{5}{3}\sqrt{6}\,i$ (complex)

An Eigenvector for λ_1 : $\vec{\mathbf{w}}_1 = \begin{bmatrix} 1 \\ 2 + \sqrt{6}\,i \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + i \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix}$

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General solutions:

$$\begin{aligned} \begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \quad &+ \quad C_1 e^{-5t} \left\{ \cos\left(\tfrac{5\sqrt{6}}{3} t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \sin\left(\tfrac{5\sqrt{6}}{3} t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\} \\ &+ \quad C_2 e^{-5t} \left\{ \sin\left(\tfrac{5\sqrt{6}}{3} t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \cos\left(\tfrac{5\sqrt{6}}{3} t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\} \end{aligned}$$

Example 2. (continued. Initial Condition.)

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$$\begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + C_1 e^{-5t} \left\{ \cos\left(\tfrac{5\sqrt{6}}{3} t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \sin\left(\tfrac{5\sqrt{6}}{3} t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\}$$

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Initial Condition:

$$\begin{bmatrix} I_1(0) \\ I_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + C_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \Rightarrow \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} -2 \\ \frac{2}{3}\sqrt{6} \end{bmatrix}$$

Example 2. (continued. Initial Condition.)

General solutions:

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$$\quad + \quad C_2 e^{-5t} \left\{ \sin\left(\frac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \cos\left(\frac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\}$$

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Solution of the initial value problem:

$$\begin{aligned} I_1(t) &= 2 - 2e^{-5t} \cos\left(\frac{5\sqrt{6}}{3}t\right) + \frac{2}{3}\sqrt{6}e^{-5t} \sin\left(\frac{5\sqrt{6}}{3}t\right) \\ I_2(t) &= \frac{10}{3}\sqrt{6}e^{-5t} \sin\left(\frac{5\sqrt{6}}{3}t\right) \end{aligned}$$

Example 2. (continued. Initial Condition.)

General solutions:

$$\begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + C_1 e^{-5t} \left\{ \cos\left(\frac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \sin\left(\frac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\}$$
$$+ C_2 e^{-5t} \left\{ \sin\left(\frac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \cos\left(\frac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\}$$

Initial Condition:

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Solution of the initial value problem:

$$\begin{aligned} I_1(t) &= 2 - 2e^{-5t}\cos\left(\frac{5\sqrt{6}}{3}t\right) + \frac{2}{3}\sqrt{6}e^{-5t}\sin\left(\frac{5\sqrt{6}}{3}t\right) \\ I_2(t) &= \frac{10}{3}\sqrt{6}e^{-5t}\sin\left(\frac{5\sqrt{6}}{3}t\right) \\ I_3(t) &= I_1(t) + I_2(t) = 2 - 2e^{-5t}\cos\left(\frac{5\sqrt{6}}{3}t\right) + 4\sqrt{6}e^{-5t}\sin\left(\frac{5\sqrt{6}}{3}t\right) \end{aligned}$$

Example 2. (continued. Figures.)

General solutions:

$$\begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + C_1 e^{-5t} \left\{ \cos\left(\frac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \sin\left(\frac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\}$$

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Example 2. (continued. Figures.)

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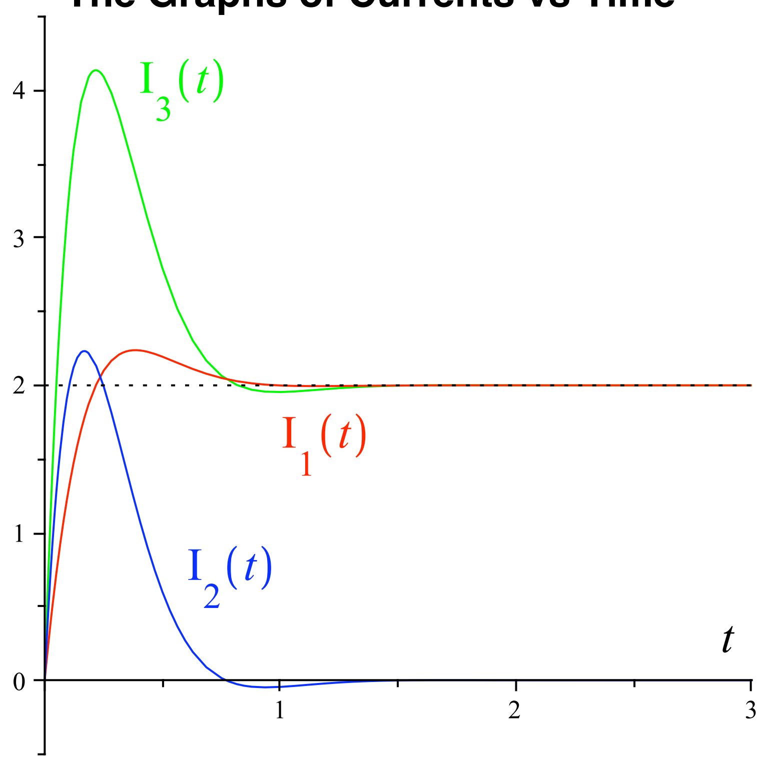
$$\quad + \quad C_2 e^{-5t} \left\{ \sin\left(\tfrac{5\sqrt{6}}{3} t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \cos\left(\tfrac{5\sqrt{6}}{3} t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\}$$

The Graphs of Currents vs Time



Example 2. (continued. Figures.)

General solutions:

$$\begin{bmatrix} I_1(t) \\ I_2(t) \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + C_1 e^{-5t} \left\{ \cos\left(\tfrac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \sin\left(\tfrac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\}$$

$$+ C_2 e^{-5t} \left\{ \sin\left(\tfrac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \cos\left(\tfrac{5\sqrt{6}}{3}t\right) \begin{bmatrix} 0 \\ \sqrt{6} \end{bmatrix} \right\}$$