Course Topics of MATH 2552

- Solution Formulas of Diff Eqs
- Phase Portraits and Stability/Instability
- Laplace Transforms
- Nonlinear Systems
- Real World Problems
- Numerical Methods

$$\frac{dy}{dt} = ry$$

A Simple but Important Diff Eq.

Constant Growth Rate $= r$.

General solutions: $y(t) = Ce^{rt}$

Initial Value Problem

$$\frac{dy}{dt} = ry, \qquad y(t_0) = y_0$$

The unique solution: $y(t) = y_0 e^{r(t-t_0)}$

- $\frac{dy}{dt} + a(t)y = b(t)$

  Solved by integrating factor

- $\frac{dy}{dt} = f(y)g(t)$

  Solved by separating variables

- $\frac{dy}{dt} = f(y)$

  - Solved by separating variables.
  - Equilibria.
  - Phase portraits.
  - Stability, asymptotic stability, and instability.

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x}$$

$$\frac{d\mathbf{x}}{dt} = A(\mathbf{x} - \mathbf{a})$$

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + \mathbf{b}$$

- **n-D Linear Systems with constant coefficients:** solved by eigenvalues, eigenvectors.
- **2-D Linear Systems with constant coefficients :** repeated eigenvalues and generalized eigenvectors.
- **2-D Lin Systems with const coeff:** Phase portraits.
- **n-D Lin Systems with const coeff:** Stability, asymptotic stability, and instability, in most cases, can be determined by the signs of $\mathrm{Re}(\lambda)$.

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = 0$$

- Characteristic roots (eigenvalues) ⇨ Solutions
- Eigenvalues, Eigenvectors.
- Phase portraits.
- Stability, asymptotic stability, and instability.
- Another solution method by Laplace and inverse Laplace transforms.

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = f(t)$$

- General solutions: $y = y_c + y_p$.
- Solutions by undetermined coefficients.
- Solutions by variation of parameters.
- Solutions by Laplace and inverse Laplace transforms.

$$\frac{d\mathbf{x}}{dt} = A\mathbf{x} + \mathbf{f}(t)$$

- Solutions by variation of parameters.

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = f(t)$$

Solutions by Laplace and inverse Laplace transforms.

- Table of Elementary Laplace Transforms
- Convert diff eqs to s-domain,
  by derivative formulas.
- $\mathcal{L}$ and $\mathcal{L}^{-1}$ involving discontinuous functions.
- Diff eqs with discontinuous $f(t)$.
- Transfer function, impulse response,
  free response, forced response, total response.

n-D Nonlinear Systems $\dfrac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x})$

- Equilibria, solved from $\mathbf{f}(\mathbf{x}) = \mathbf{0}$.

- The linear approximating system near an equilibrium $\mathbf{x} = \mathbf{a}$:

  $$\frac{d\mathbf{x}}{dt} = ▨\ (\mathbf{x} - \mathbf{a})$$

  where the coefficient matrix ▨ is the Jacobian matrix evaluated at equilibrium $\mathbf{x} = \mathbf{a}$.

- In many cases, the signs of the real parts of the eigenvalues of ▨

  ⇨ Stability, asymptotic stability, and instability of the equilibrium $\mathbf{x} = \mathbf{a}$ in the nonlinear system.

Real World Problems

- Newton's law of cooling/heating
- Newton's 2nd Law of Mechanics
  - Mass Rising/Falling under Gravity and Friction.
- Population dynamics
  - Malthusian Model (constant growth rate)
  - Logistic Model
  - Competing Species
  - Predator-Prey Model

Real World Problems

- Brine with Salt in Tanks
  - A Single Tank
  - Multiple Tanks
- Spring-Mass Systems

  - Free Vibrations: $my'' + \gamma y' + ky = 0$
    - No damping
    - Critical damping
    - Underdamping
    - Overdamping

  - Forced Vibrations: $my'' + \gamma y' + ky = f(t)$
    - Resonance
- Electric Circuits

$$LQ'' + RQ' + \frac{1}{C}Q = 0$$

$$Li'' + Ri' + \frac{1}{C}i = 0$$

Numerical Methods

- Euler
- Improved Euler
- Runge-Kutta