Solution Structure of
1st Order Diff Eq's

$\boxed{\text{A Single Homog. Linear Diff Eq. of 1st order}}$

$$\frac{dy}{dt} + a(t)y = 0$$

Gen. Sol's. $y(t) = C\,y_1(t)$ $\begin{cases} C: \text{a free parameter} \\ y_1(t): \text{a nonzero Sol.} \end{cases}$

Sol. Method: $\begin{cases} \bullet \text{ Integrating Factor} \\ \text{or} \\ \bullet \text{ Sep. Variables} \end{cases}$

$\boxed{\text{A Single Nonhomog. Lin. Diff Eq. of 1st order}}$

$$\frac{dy}{dt} + a(t)y = b(t).$$

Gen. Sol's. $y(t) = y_p(t) + C\,y_1(t)$

where $y_p(t)$: a particular sol of Nonhomog. Eq.

$C\,y_1(t)$: "Complementary Sol's", solutions of the corresponding Homog. Eq.

Solution Method Integrating Factor

$\boxed{\text{A Single Nonlinear Diff Eq. of 1st order}}$

$$\frac{dy}{dt} = f(t, y)$$

Gen. Sol's One free parameter C.

Solution Method.
- No general sol method.
- If $f(t, y)$ is separable, can be solved by sep. variables

Examples

- $\frac{dy}{dt} - 3y = 0$, $y(t) = Ce^{3t}$

- $\frac{dy}{dt} - 3y = 10\cos(4t)$, $y(t) = \underbrace{-\frac{6}{5}\cos(4t) + \frac{8}{5}\sin(4t)}_{y_p(t)} + \underbrace{Ce^{3t}}_{y_c(t)}$

- $\frac{dy}{dt} - 6ty = 0$, $y(t) = Ce^{3t^2}$

- $\frac{dy}{dt} - 6ty = 24te^{4t^2}$, $y(t) = \underbrace{12e^{4t^2}}_{y_p(t)} + \underbrace{Ce^{3t^2}}_{y_c(t)}$

- $\frac{dy}{dt} = 3y\left(1 - \frac{y}{4}\right)$, $y(t) = \frac{4Ce^{3t}}{1+Ce^{3t}}$

- $\frac{dy}{dt} = e^{-y} + \frac{1}{2t}$ $(t>0)$, $y(t) = \ln\left(2t + C\sqrt{t}\right)$

Example (i) Consider $\frac{dy}{dt} + a(t)y = 0$ (1st order homogeneous linear diff. eq.)

(ii) Suppose: $y_1(t) = e^{2t-t^2}$ is a particular sol.

(iii) Give three more solutions.

(ii) $\Rightarrow$ $8e^{2t-t^2}$, $\sqrt{3}e^{2t-t^2}$, $-7e^{2t-t^2}$, ...

(iv) Give the general solutions.

(ii) $\Rightarrow$ $y(t) = Ce^{2t-t^2}$

where C is an arbitrary const.

Example (i) Consider $\dfrac{dy}{dt} + a(t)y = b(t)$ (*) (1st order Nonhomogeneous linear diff eq)

(ii) Suppose: $y_p(t) = \cos(3t)$ is a particular sol. of (*).

(iii) Give another sol. of (*).

 can't

(iv) Suppose further: the homogeneous eq

$$\frac{dy_c}{dt} + a(t)y_c = 0 \qquad (*)_h$$

has a particular sol. $y_1(t) = e^{-2t^2}$

(v) Give the general sol's of $(*)_h$.

$$y_c(t) = Ce^{-2t^2}$$

(vi) Give the general sol's of (*).

$$y(t) = y_p(t) + y_c(t)$$
$$= \cos(3t) + Ce^{-2t^2}$$

Example (i) Consider $\frac{dy}{dt} = f(t,y)$ (nonlin. diff eq.)

(ii) Suppose:

$y_1(t) = e^{2t-t^2}$, $y_2(t) = 3e^{t^2}$, $y_3(t) = 4e^{3t^2}$, $y_4 = 7e^{3t^2}$

are solutions

(iii) Give more solutions:

 Can't