• Diff Eq $\frac{dy}{dt} = f(t, y)$

① For both lin. & nonlin. eq's

if $f(t, y)$ has discontinuities,

solutions $y(t)$ may not exist beyond the discontinuities of f

& the t-intervals of existence become limited.

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② For some nonlin. eq's

even if $f(t, y)$ is continuous & smooth everywhere,

solutions $y(t)$ may not exist beyond some limited intervals.

This does not occur to lin. diff eq's.

Diff Eq $\frac{dy}{dt} = \frac{t-y}{t-7}$ $f(t,y) = \frac{t-y}{t-7}$ is discontinuous at $t = 7$.

$$\frac{dy}{dt} = \frac{t-y}{t-7}$$

$y(-6) = 2$

Find the maximal interval of existence of the solution. **Answer:**

$$\frac{dy}{dt} = \frac{t-y}{t-7}$$

$y(11) = 10$

Find the maximal interval of existence of the solution. **Answer:**

$$\frac{dy}{dt} = \frac{t-y}{t-7}$$

$y(4) = -8$

Find the maximal interval of existence of the solution. **Answer:**

$$\frac{dy}{dt} = \frac{t-y}{t-7}$$

$y(12) = -5$

Find the maximal interval of existence of the solution. **Answer:**

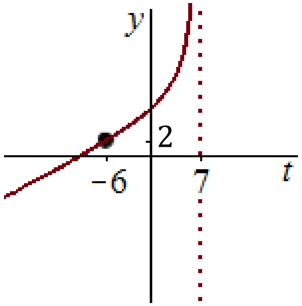
Diff Eq $\frac{dy}{dt} = \frac{t-y}{t-7}$ $f(t,y) = \frac{t-y}{t-7}$ is discontinuous at $t = 7$.

Gen Sols $y(t) = \frac{t^2 + C}{2(t-7)}$

$\frac{dy}{dt} = \frac{t-y}{t-7}$

$y(-6) = 2$

Find the maximal interval of existence of the solution.

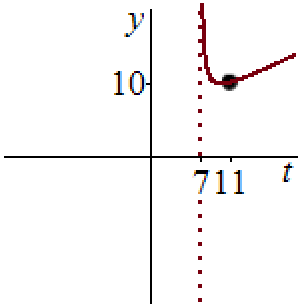


$y(t) = \frac{t^2-88}{2(t-7)}$ $(-\infty < t < 7)$

$\frac{dy}{dt} = \frac{t-y}{t-7}$

$y(11) = 10$

Find the maximal interval of existence of the solution.

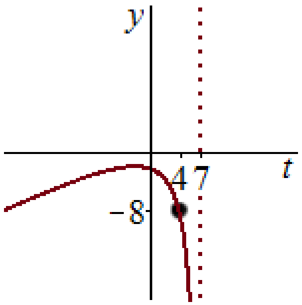


$y(t) = \frac{t^2-41}{2(t-7)}$ $(7 < t < \infty)$

$\frac{dy}{dt} = \frac{t-y}{t-7}$

$y(4) = -8$

Find the maximal interval of existence of the solution.

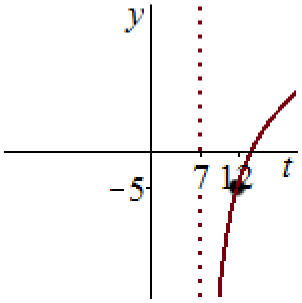


$y(t) = \frac{t^2+32}{2(t-7)}$ $(-\infty < t < 7)$

$\frac{dy}{dt} = \frac{t-y}{t-7}$

$y(12) = -5$

Find the maximal interval of existence of the solution.



$y(t) = \frac{t^2-97}{2(t-7)}$ $(7 < t < \infty)$

Diff Eq $$(t+5)^{2/3}\frac{dy}{dt}+\left(\frac{1}{3}\cos\frac{t}{2}\right)y=\frac{20}{(t-8)^2} \iff \frac{dy}{dt}=\frac{20}{(t+5)^{2/3}(t-8)^2}-\frac{1}{(t+5)^{2/3}}\left(\frac{1}{3}\cos\frac{t}{2}\right)y$$

$$f(t,y)=\frac{20}{(t+5)^{2/3}(t-8)^2}-\frac{1}{(t+5)^{2/3}}\left(\frac{1}{3}\cos\frac{t}{2}\right)y$$

is discontinuous at $t=-5$, and $t=8$.

$$\frac{dy}{dt}=f(t,y)$$

$y(3)=4$

Find the maximal interval of existence of the solution. **Answer:**

$$\frac{dy}{dt}=f(t,y)$$

$y(-9)=-4$

Find the maximal interval of existence of the solution. **Answer:**

$$\frac{dy}{dt}=f(t,y)$$

$y(12)=10$

Find the maximal interval of existence of the solution. **Answer:**

$$\frac{dy}{dt}=f(t,y)$$

$y(5)=-8$

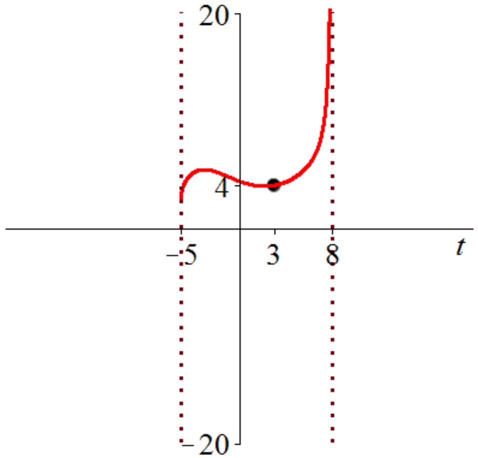
Find the maximal interval of existence of the solution. **Answer:**

Diff Eq $\quad (t+5)^{2/3}\dfrac{dy}{dt}+\left(\dfrac{1}{3}\cos\dfrac{t}{2}\right)y=\dfrac{20}{(t-8)^2} \iff \dfrac{dy}{dt}=\dfrac{20}{(t+5)^{2/3}(t-8)^2}-\dfrac{1}{(t+5)^{2/3}}\left(\dfrac{1}{3}\cos\dfrac{t}{2}\right)y$

$f(t,y)=\dfrac{20}{(t+5)^{2/3}(t-8)^2}-\dfrac{1}{(t+5)^{2/3}}\left(\dfrac{1}{3}\cos\dfrac{t}{2}\right)y$ is discontinuous at $t=-5$, and $t=8$.

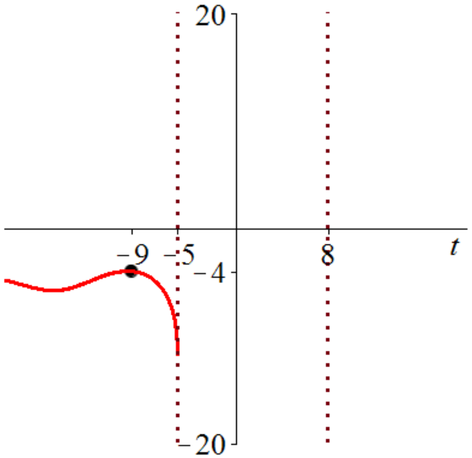
$\dfrac{dy}{dt}=f(t,y)$

$y(3)=4$



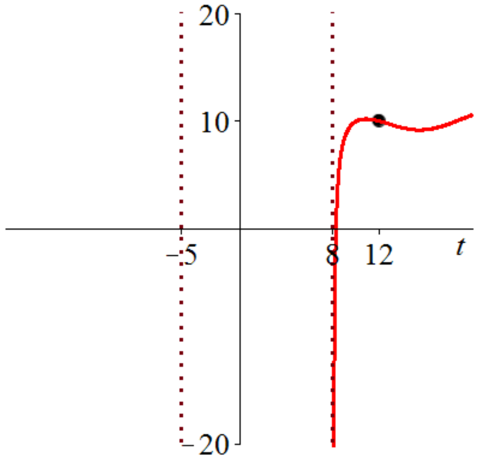
$\dfrac{dy}{dt}=f(t,y)$

$y(-9)=-4$



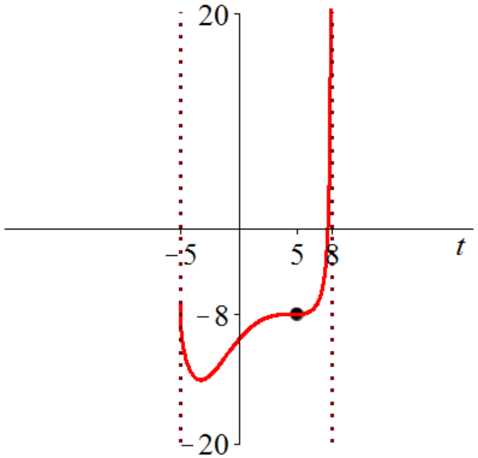
$\dfrac{dy}{dt}=f(t,y)$

$y(12)=10$



$\dfrac{dy}{dt}=f(t,y)$

$y(5)=-8$



Intervals of existence of solutions of nonlinear diff eqs can be more difficult.

Example $\frac{dy}{dt} = \frac{y^2}{6}$ $f(t,y) = \frac{y^2}{6}$ is continuous everywhere.

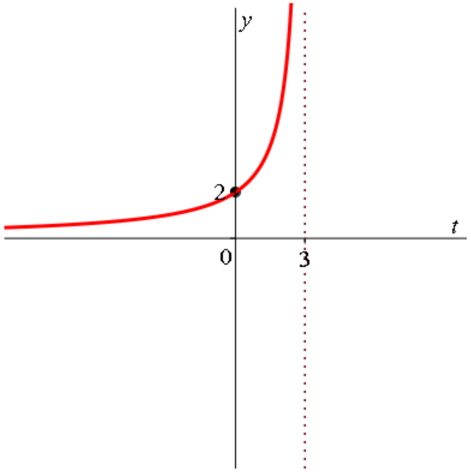
Gen Sols $y(t) = \frac{-6}{t+C}$

$$\frac{dy}{dt} = \frac{y^2}{6}$$

$$y(0) = 2$$

$$y(t) = \frac{-6}{t-3}$$

for $t \in (-\infty, 3)$

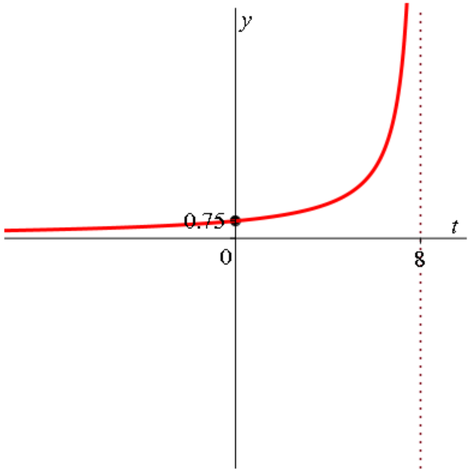


$$\frac{dy}{dt} = \frac{y^2}{6}$$

$$y(0) = \frac{3}{4}$$

$$y(t) = \frac{-6}{t-8}$$

for $t \in (-\infty, 8)$

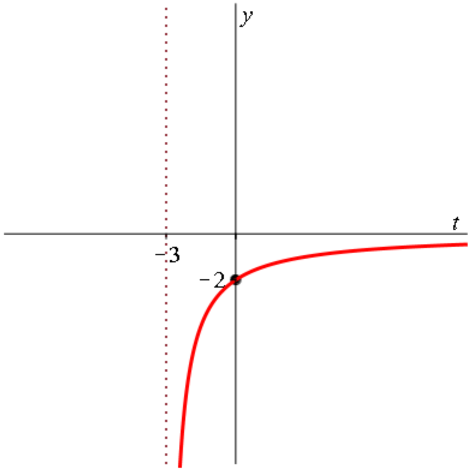


$$\frac{dy}{dt} = \frac{y^2}{6}$$

$$y(0) = -2$$

$$y(t) = \frac{-6}{t+3}$$

for $t \in (-3, \infty)$



$$\frac{dy}{dt} = \frac{y^2}{6}$$

$$y(0) = \frac{3}{4}$$

$$y(t) = \frac{-6}{t+8}$$

for $t \in (-8, \infty)$