Intervals of Existence of Solutions to Differential Equations

Diff Eq $\frac{dy}{dt} = f(t, y)$

① For both linear & nonlinear diff eq's:

if $f(t, y)$ has discontinuities,
solutions $y(t)$ may not exist beyond the discontinuities of f
& the t–intervals of existence become limited.

② For some nonlinear diff eq's:

even if $f(t, y)$ is continuous & smooth everywhere,
solutions $y(t)$ may not exist beyond some limited intervals.

Remark: This does not occur for linear diff eq's.

Diff Eq $\frac{dy}{dt} = \frac{t-y}{t-7}$ $f(t,y) = \frac{t-y}{t-7}$ is discontinuous at $t = 7$.

$$\frac{dy}{dt} = \frac{t-y}{t-7}$$

$y(-6) = 2$

Find the maximal interval of existence of the solution. **Answer:**

$$\frac{dy}{dt} = \frac{t-y}{t-7}$$

$y(11) = 10$

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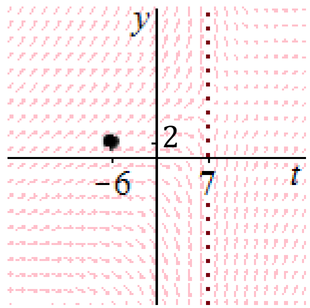

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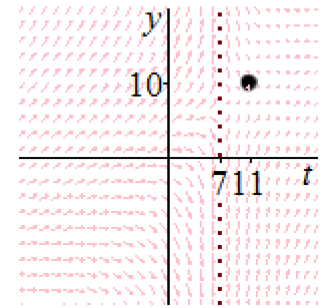

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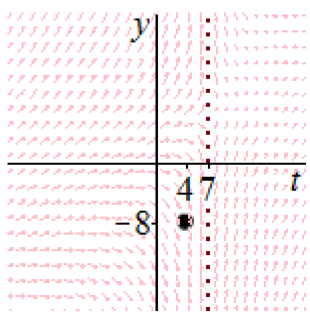

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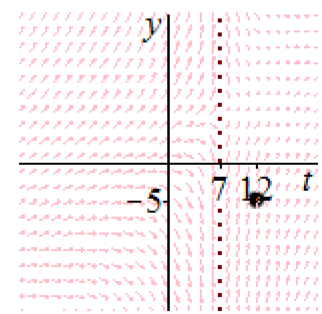

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Diff Eq $\dfrac{dy}{dt} = \dfrac{t-y}{t-7}$ $\quad f(t,y) = \dfrac{t-y}{t-7}$ is discontinuous at $t = 7$.

Gen Sols $y(t) = \dfrac{t^2 + C}{2(t-7)}$

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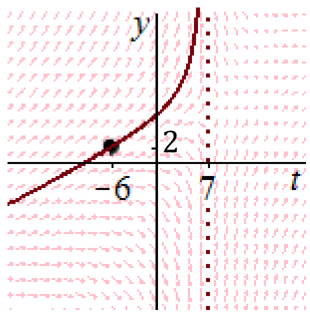

Find the maximal interval of existence of the solution.

$y(t) = \dfrac{t^2-88}{2(t-7)}$ $(-\infty < t < 7)$

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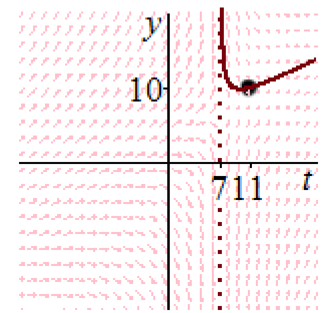

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$y(t) = \dfrac{t^2-41}{2(t-7)}$ $(7 < t < \infty)$

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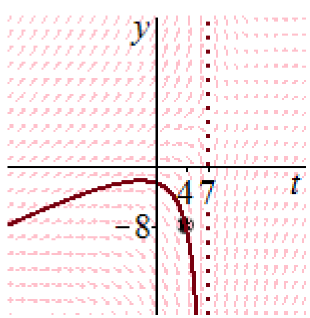

Find the maximal interval of existence of the solution.

$y(t) = \dfrac{t^2+32}{2(t-7)}$ $(-\infty < t < 7)$

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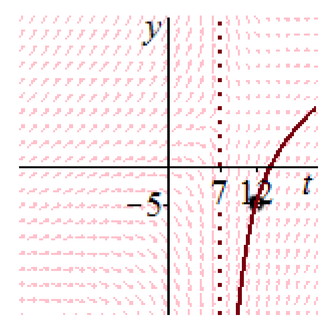

Find the maximal interval of existence of the solution.

$y(t) = \dfrac{t^2-97}{2(t-7)}$ $(7 < t < \infty)$

Diff Eq $(t+5)^{2/3}\frac{dy}{dt}+\left(\frac{1}{3}\cos\frac{t}{2}\right)y=\frac{20}{(t-8)^2} \iff \frac{dy}{dt}=\frac{20}{(t+5)^{2/3}(t-8)^2}-\frac{1}{(t+5)^{2/3}}\left(\frac{1}{3}\cos\frac{t}{2}\right)y$

$f(t,y)=\frac{20}{(t+5)^{2/3}(t-8)^2}-\frac{1}{(t+5)^{2/3}}\left(\frac{1}{3}\cos\frac{t}{2}\right)y$ is discontinuous at $t=-5$, and $t=8$.

$$\frac{dy}{dt}=f(t,y)$$

$y(3)=4$

Find the maximal interval of existence of the solution. **Answer:**

$$\frac{dy}{dt}=f(t,y)$$

$y(-9)=-4$

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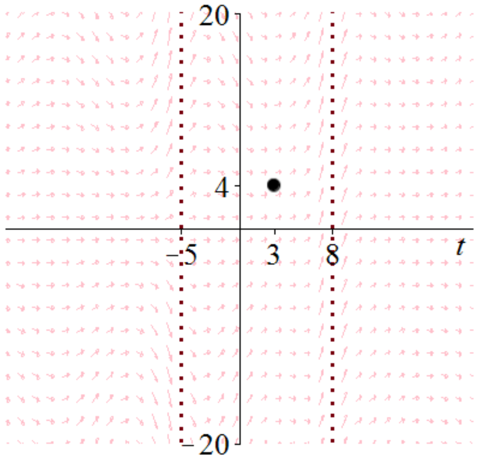

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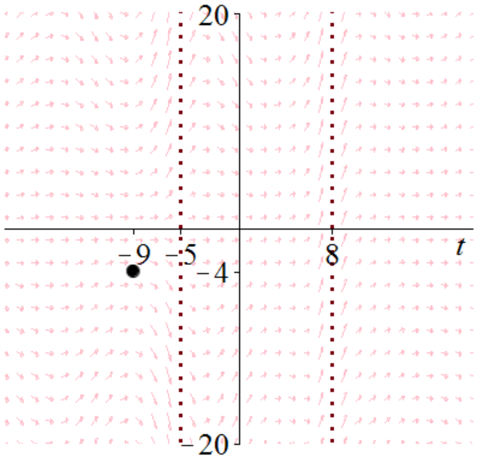

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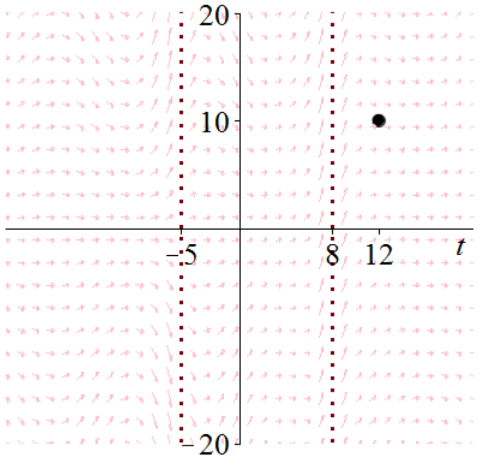

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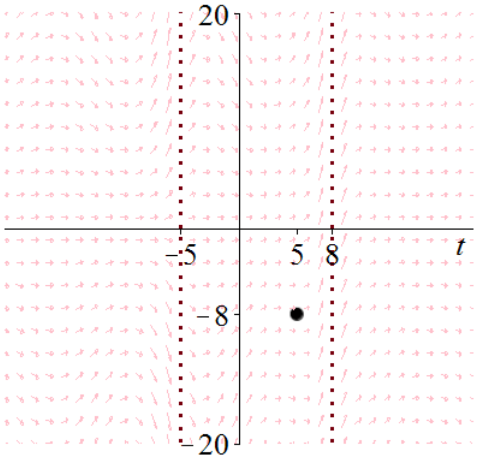

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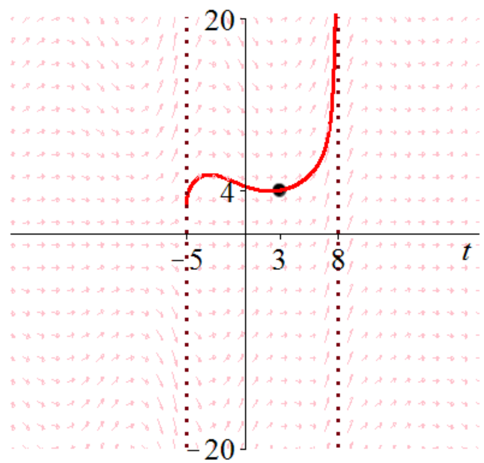

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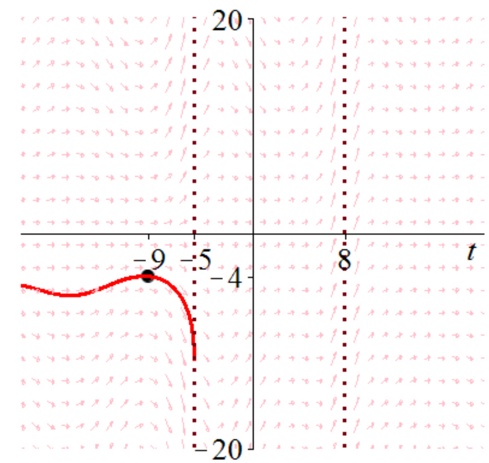

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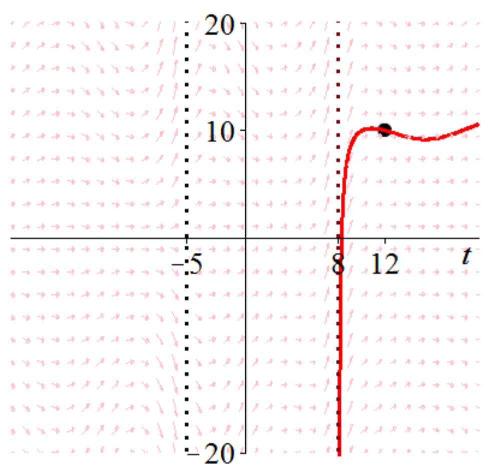

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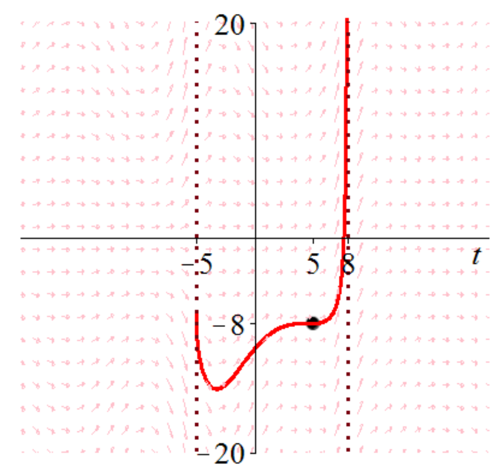


Intervals of existence of solutions of nonlinear diff eqs can be more difficult.

Example $\frac{dy}{dt} = \frac{y^2}{6}$ $\quad f(t,y) = \frac{y^2}{6}$ is continuous everywhere.

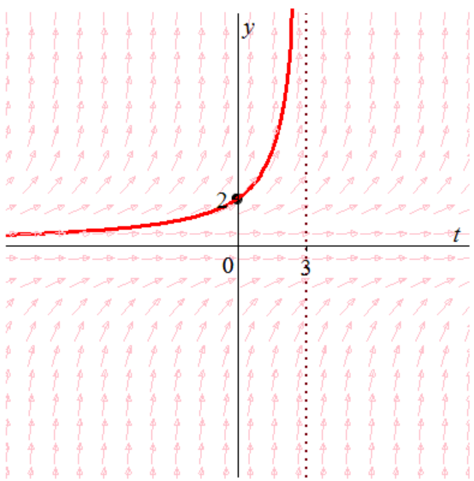
Gen Sols $\quad y(t) = \frac{-6}{t + C}$

$$\frac{dy}{dt} = \frac{y^2}{6}$$

$$y(0) = 2$$

$$y(t) = \frac{-6}{t-3}$$

for $t \in (-\infty, 3)$

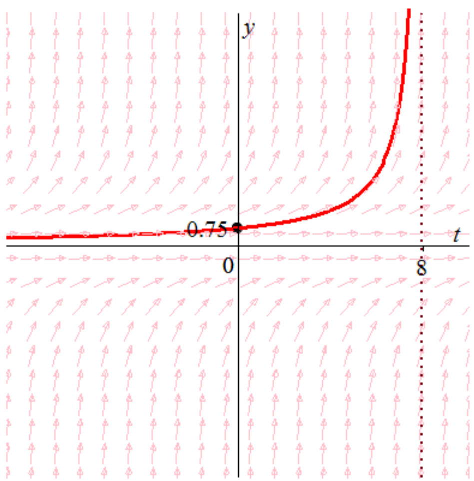


$$\frac{dy}{dt} = \frac{y^2}{6}$$

$$y(0) = \frac{3}{4}$$

$$y(t) = \frac{-6}{t-8}$$

for $t \in (-\infty, 8)$

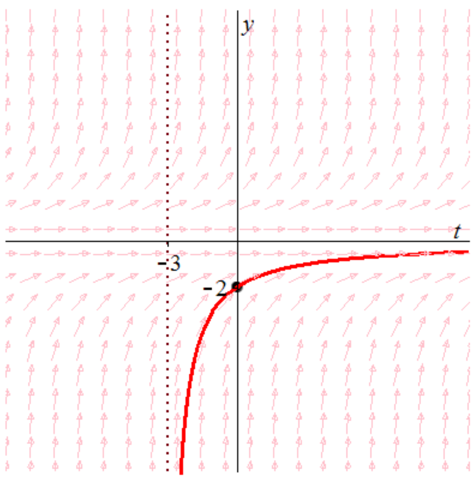


$$\frac{dy}{dt} = \frac{y^2}{6}$$

$$y(0) = -2$$

$$y(t) = \frac{-6}{t+3}$$

for $t \in (-3, \infty)$



$$\frac{dy}{dt} = \frac{y^2}{6}$$

$$y(0) = -\frac{3}{4}$$

$$y(t) = \frac{-6}{t+8}$$

for $t \in (-8, \infty)$