Phase Portraits

Obtain the solution behavior **without** solving the diff eq's.

Example $P' = 0.03\left(1 - \frac{P}{4}\right)P = f(P)$.

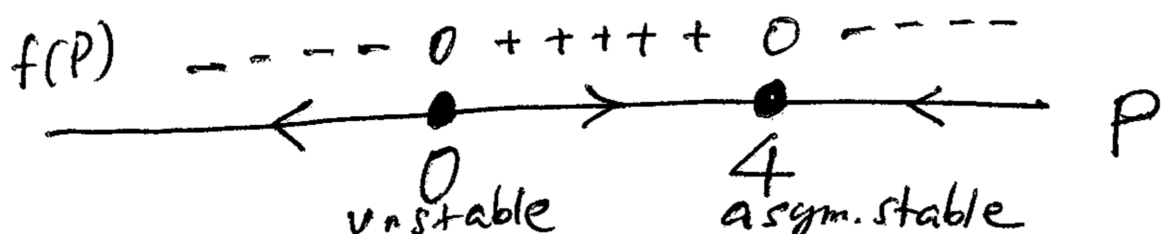
Equilibria $\Leftrightarrow P' = 0 \Leftrightarrow 0.03\left(1-\frac{P}{4}\right)P = 0$

$P = 0,\ 4$

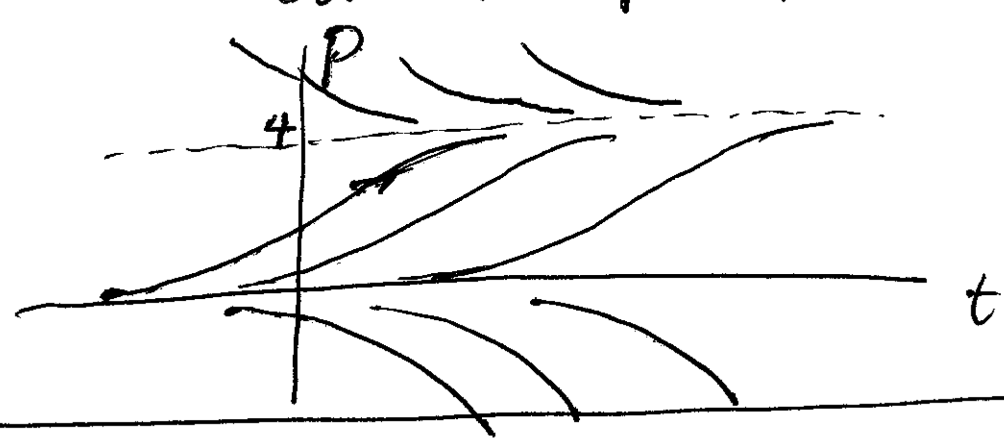
Increasing Sol's $\Leftrightarrow P' > 0 \Leftrightarrow f(P) > 0$

Decreasing Sol's $\Leftrightarrow P' < 0 \Leftrightarrow f(P) < 0$

Phase Portrait



Solution Graphs P(t) vs t



General Algorithms for $y' = f(y)$.

- Find Equilibria $\Leftrightarrow$ $y' = f(y) = 0$.
- For Non-Equilibrium Sol's.
  Determine whether $y(t)$ increases or decreases
  $\Leftrightarrow$ Look at the sign of $y' = f(y)$
- Sketch the Phase Portrait
- Read off stability, asym. stability or instability from phase portrait.

Example $y' = (e^y - 1)(y-2)^2(y+1)$

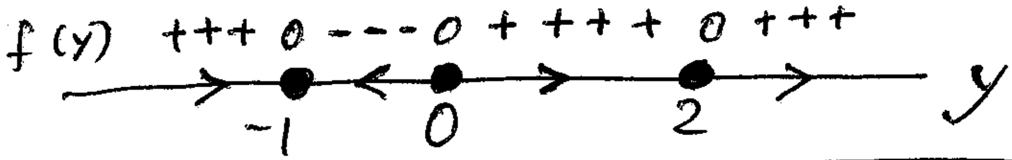
(a) Find equilibria

(b) Determine their stability / instability.

Equilibria $\Leftrightarrow y' = 0 \Leftrightarrow f(y) = 0$

$y = 0, \; y = 2, \; y = -1.$

Phase Portrait

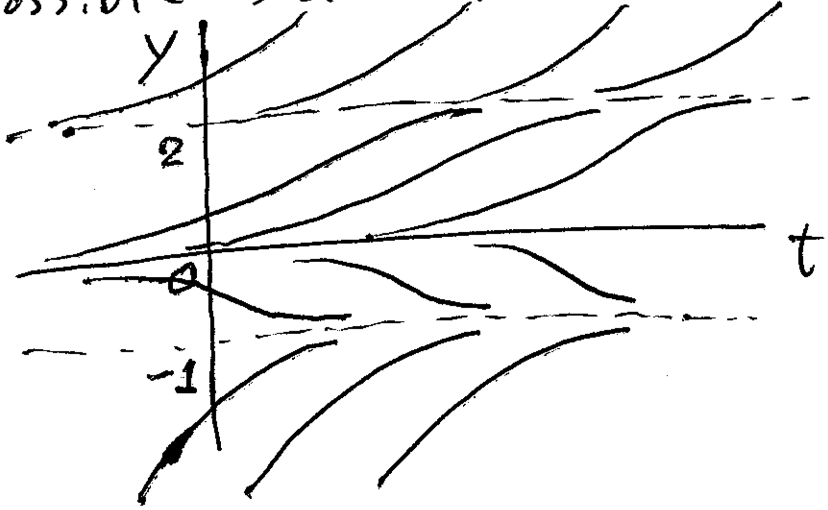


$y = -1$ is asymp. stable

$y = 0$ is unstable

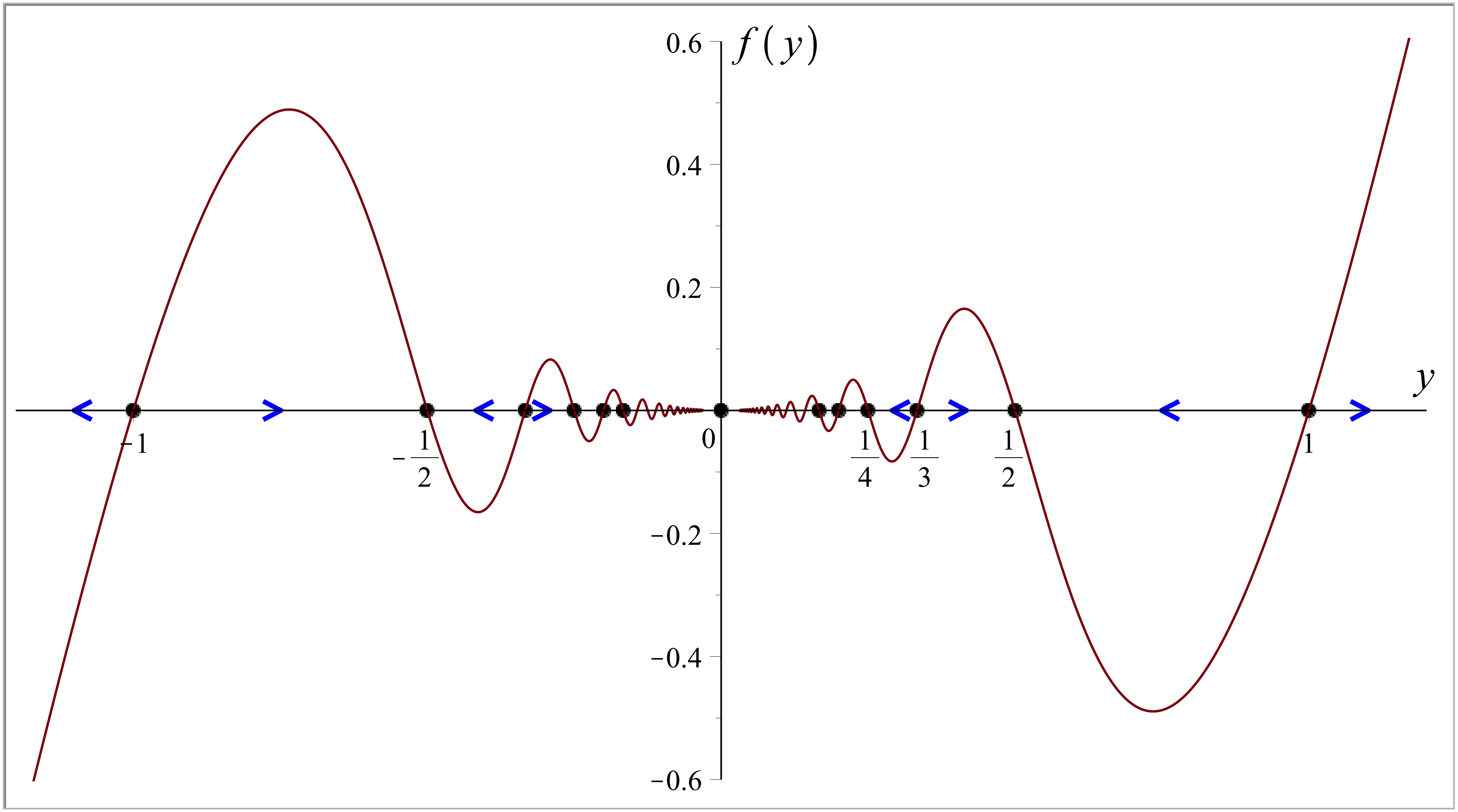
$y = 2$ is semi-stable (& is unstable)

Possible Sol. Graphs $y(t)$ vs t



EXAMPLE: $\dfrac{dy}{dt} = f(y)$ where $f(y) = \begin{cases} y^2 \sin\left(\dfrac{\pi}{y}\right) & y \neq 0 \\ 0 & y = 0 \end{cases}$

Diff Eq has infinitely many equilibria near 0: $y = \pm 1, \ \pm\dfrac{1}{2}, \ \pm\dfrac{1}{3}, \ \pm\dfrac{1}{4}, \cdots$



Equilibrium $y = 0$ is stable, but **not** asymptotically stable.