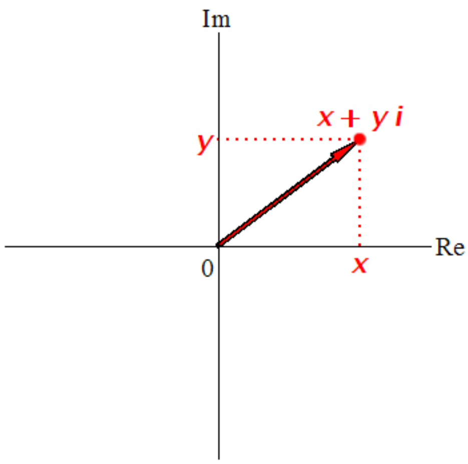
Complex Plane ---Geometry of Complex Numbers

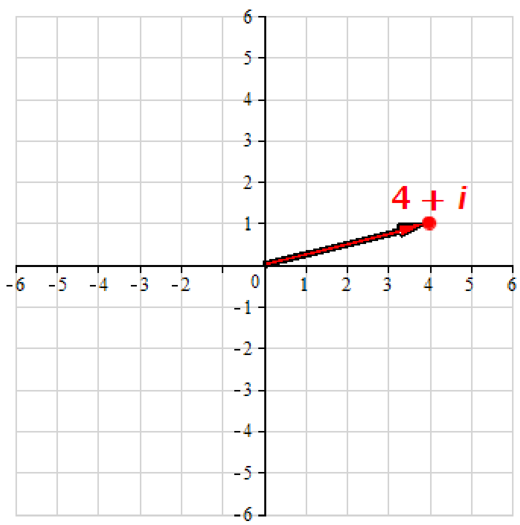


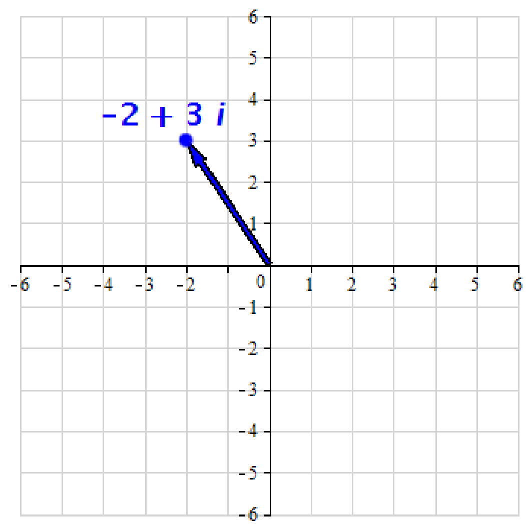
$z = x + yi$ (with x and y real)

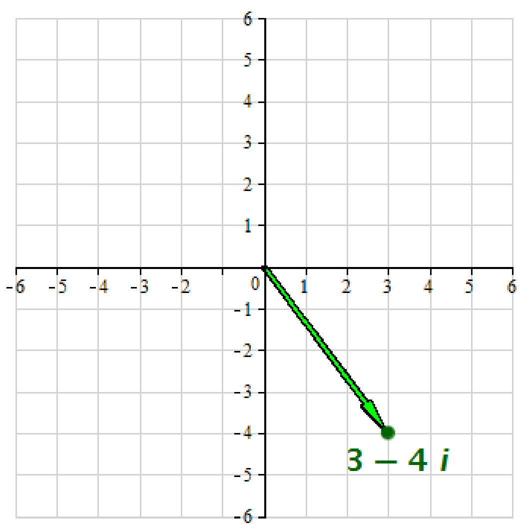
$\Leftrightarrow$ (x, y)

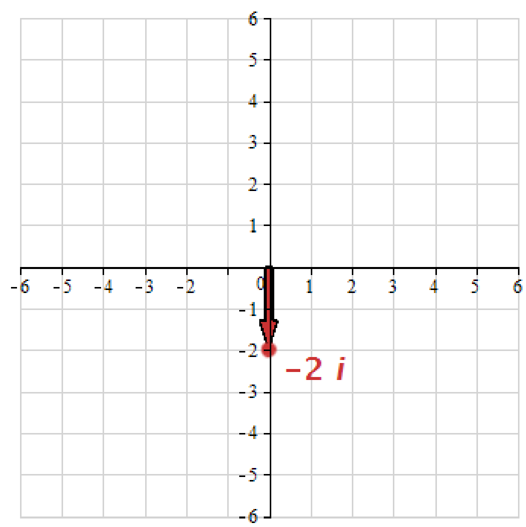
$\Leftrightarrow$ a point on the coordinate plane

$\Leftrightarrow$ a vector on the plane



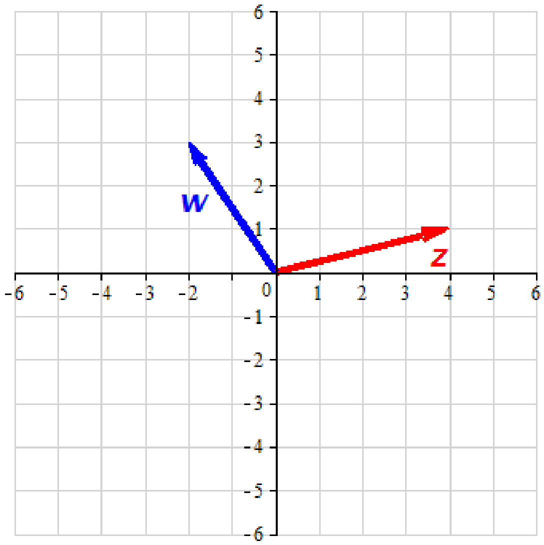




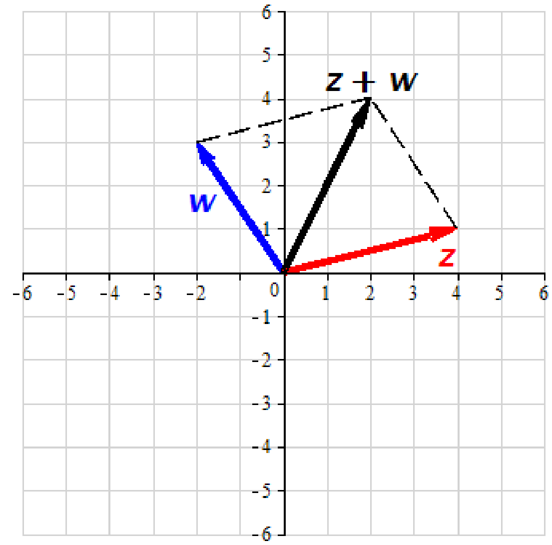


Geometry of Addition

$z = 4 + i,\ w = -2 + 3i$



$z = 4 + i,\ w = -2 + 3i,\ z + w = 2 + 4i$



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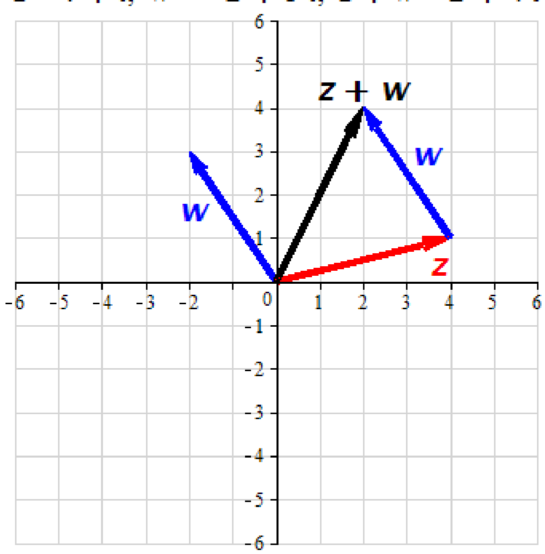

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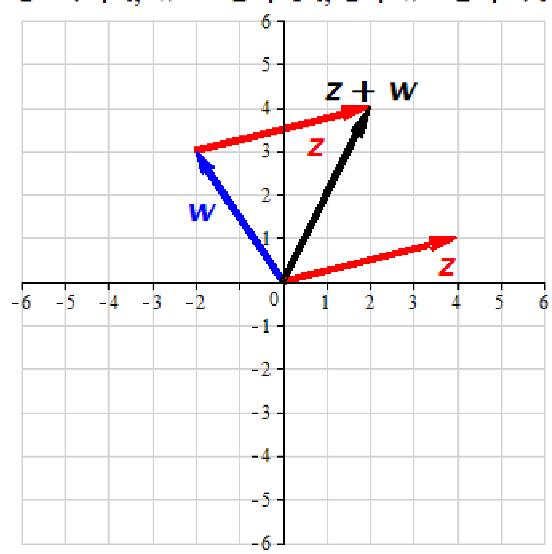
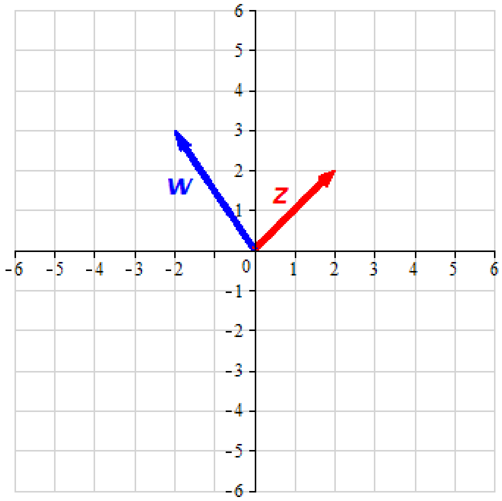
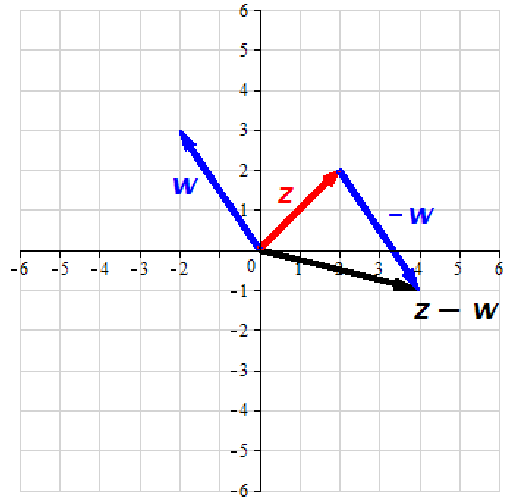


Geometry of Subtraction

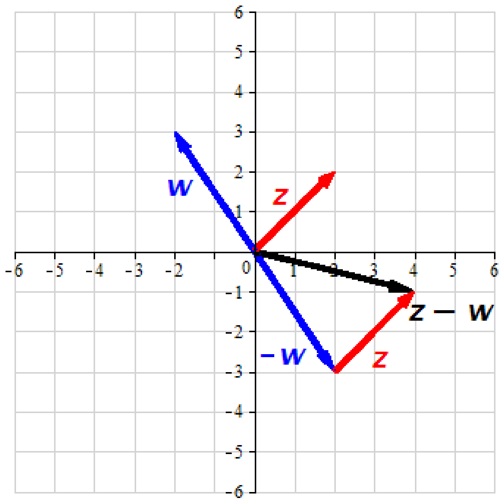
$z = 2 + 2i,\ \ w = -2 + 3i$



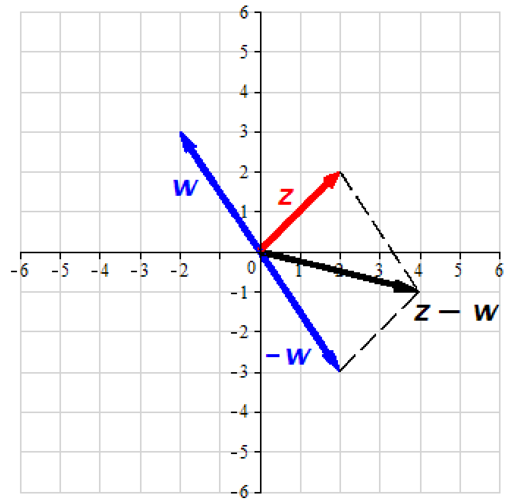
$z - w = 4 - i$



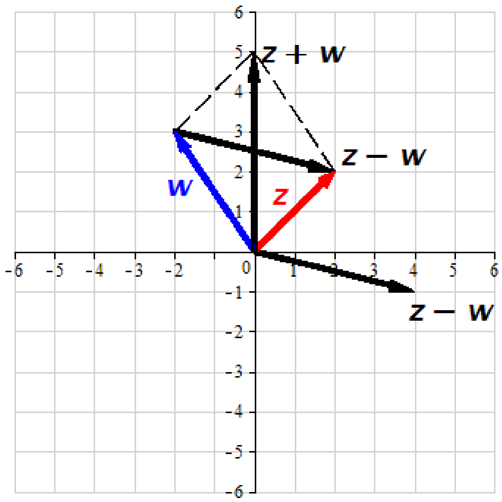
$z - w = 4 - i$



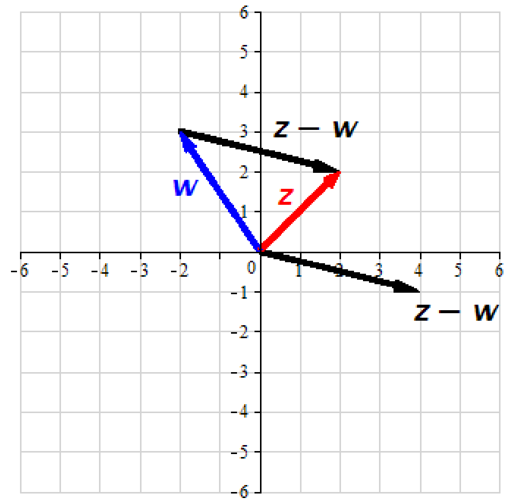
$z - w = 4 - i$



$z - w = 4 - i$

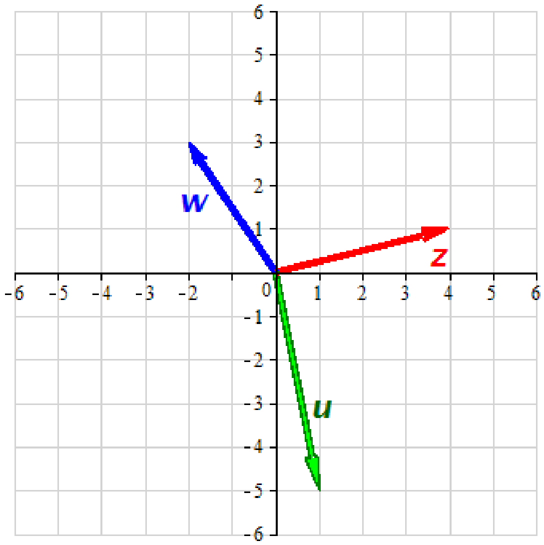


$z - w = 4 - i$

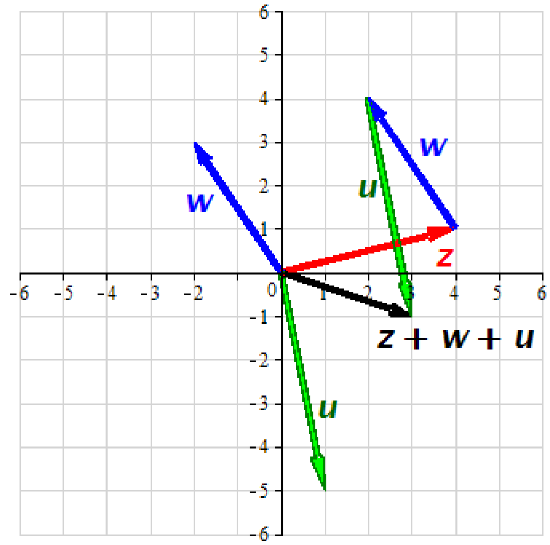


Examples Involve More Than Two Vectors

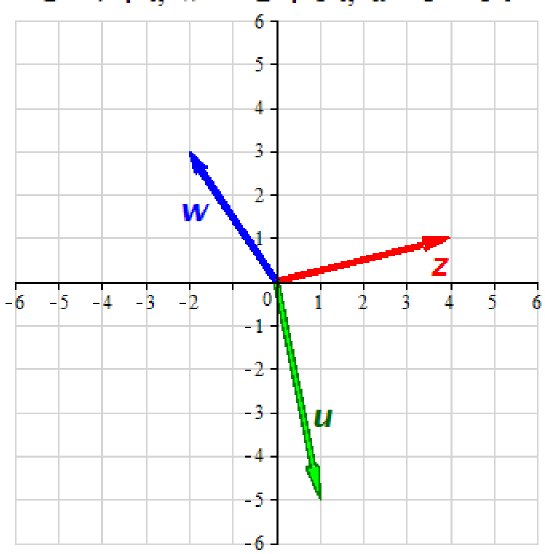
$z = 4 + i,\ w = -2 + 3i,\ u = 1 - 5i$



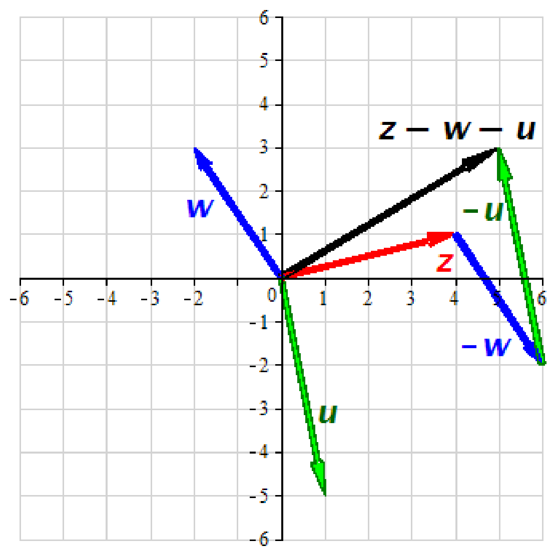
$z + w + u = 3 - i$



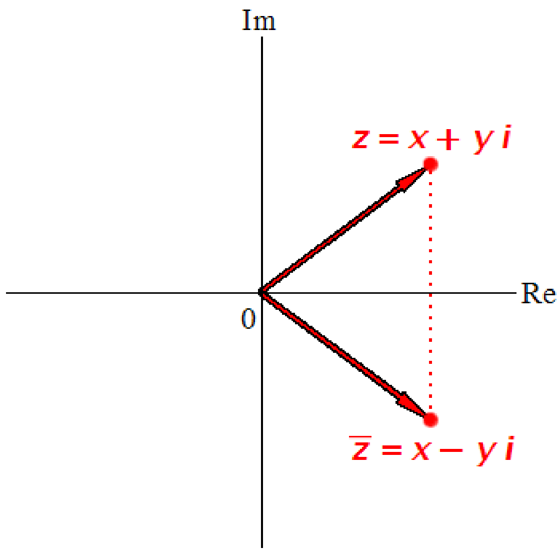
$z = 4 + i,\ w = -2 + 3i,\ u = 1 - 5i$



$z - w - u = 5 + 3i$



More Geometry: Conjugate, Modulus, Distance.



The complex conjugate $\bar{z}$

= the reflection of z

about the x-axis.

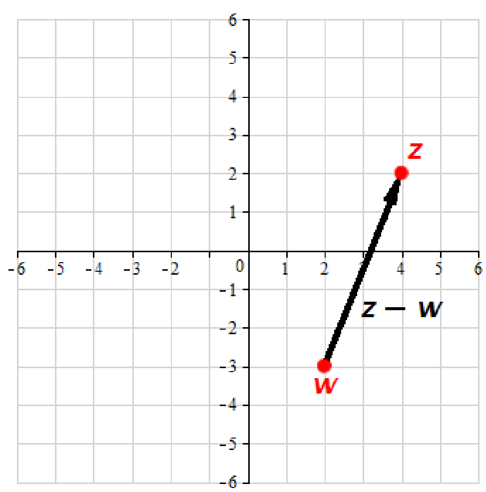
$|z| = \sqrt{x^2 + y^2}$ is called the **absolute value** of z, or, the **modulus** of z.

$|z|$ = the distance from the origin to point z

= the length of vector z.

$z\bar{z} = x^2 + y^2 = |z|^2$. $\quad |z_1 z_2| = |z_1|\,|z_2|$, $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$.

$|z - w|$ = the distance from w to z.



Example: $z = 4 + 2i$, $w = 2 - 3i$.

The distance between z and w is

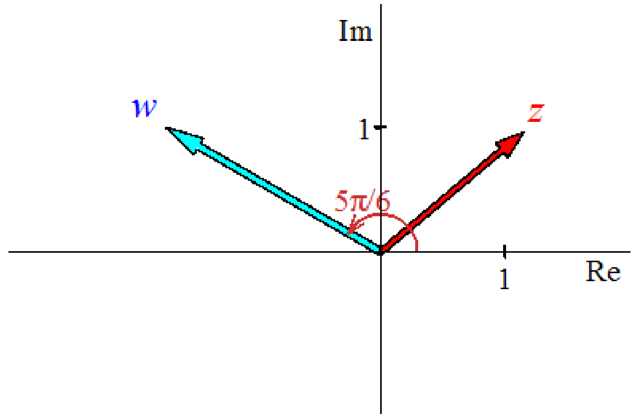
$$|z - w| = |2 + 5i|$$

$$= \sqrt{2^2 + 5^2} = \sqrt{29}$$

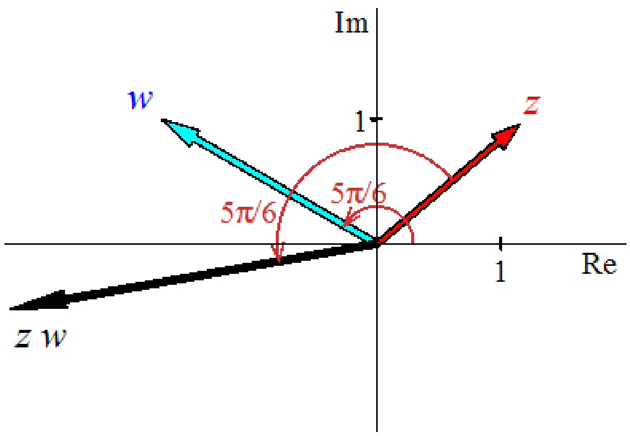
Geometry of Multiplication and Division

Example: Given z as in the figure, and $w = -\sqrt{3} + i$.

What are the geometric meanings of zw and z/w?



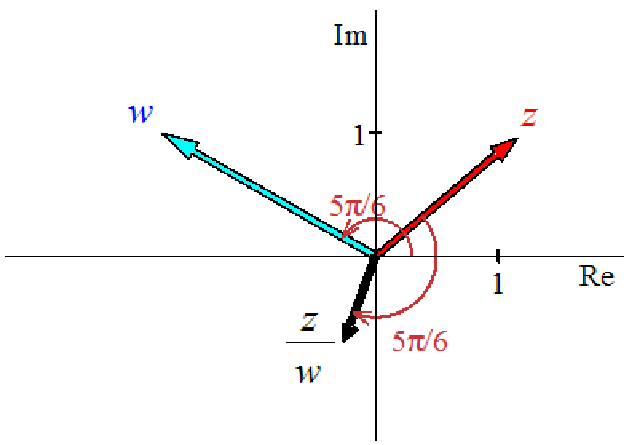
Solution: Put z and w in the polar form: $z = re^{i\theta}$, $w = 2e^{i5\pi/6}$.



The multiplication

$$zw = 2re^{i\left(\theta + \frac{5\pi}{6}\right)}$$

is to rotate z counterclockwise by angle $5\pi/6$, and then 2 times the length.



The division

$$\frac{z}{w} = \frac{r}{2} e^{i\left(\theta - \frac{5\pi}{6}\right)}$$

is to rotate z clockwise by angle $5\pi/6$, and then $\frac{1}{2}$ times the length.

In general, length of zw = (length of z)(length of w),

argument of zw = (argument of z) + (argument of w),

length of z/w = (length of z)/(length of w),

argument of z/w = (argument of z) − (argument of w).

The argument angles are uniformly spaced.

The roots are the vertices of a regular pentagon.