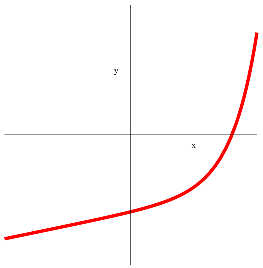
Increasing and Decreasing Functions

Xu-Yan Chen

Theorem.

If $f'(x) > 0$ on an interval (a, b),
then $f(x)$ increases on (a, b);
that is, $f(x_1) < f(x_2)$ for all $a < x_1 < x_2 < b$.



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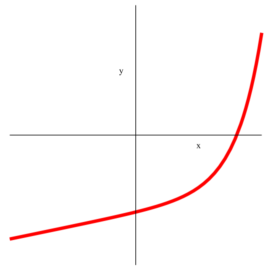

If $f'(x) < 0$ on an interval (a, b),
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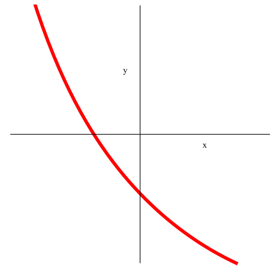

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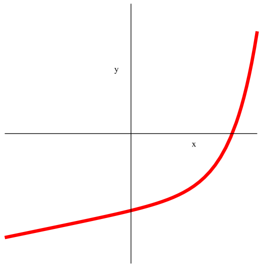

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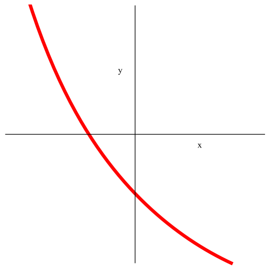
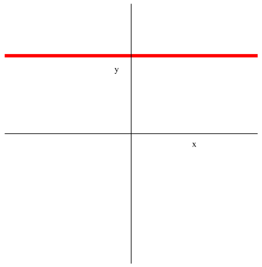


If $f'(x) = 0$ on an interval (a, b),
then $f(x)$ is constant on (a, b).



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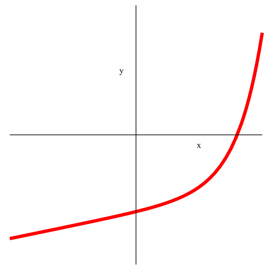

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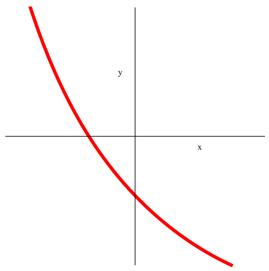

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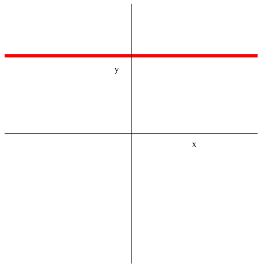

Proof. Apply the mean value theorem.

Example 1.

$$f(x) = \frac{1}{3}x^3 + x - 2$$

$f'(x) = x^2 + 1 > 0$ for all real x.

$f(x)$ increases on $(-\infty, \infty)$.

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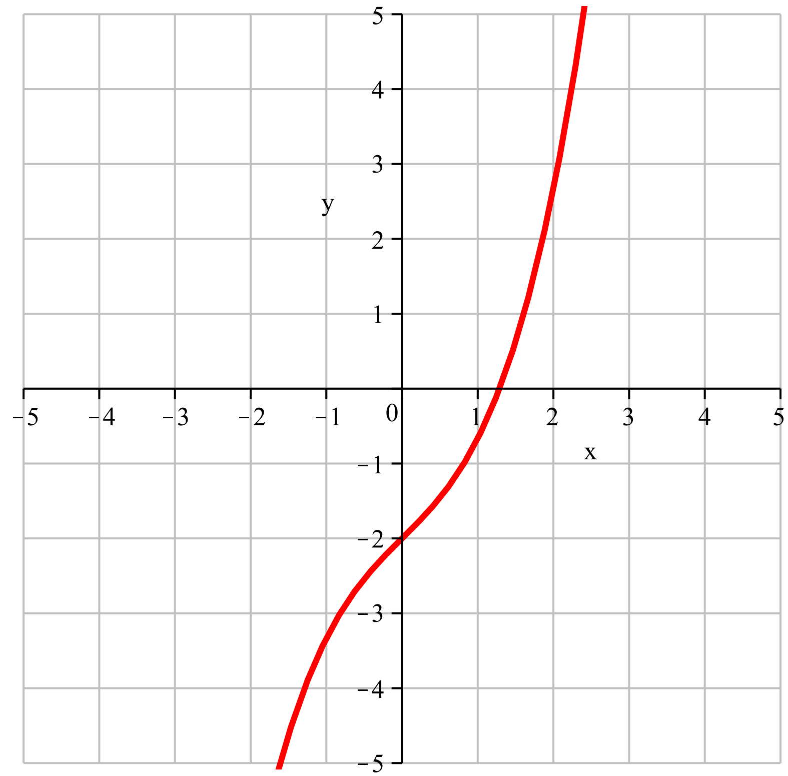

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x **interval**	$-\infty < x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$2 < x < \infty$
Sign of $f'(x)$	$-$	0	$+$	0	$-$
Behavior of $f(x)$	$\searrow$	-6	$\nearrow$	3	$\searrow$

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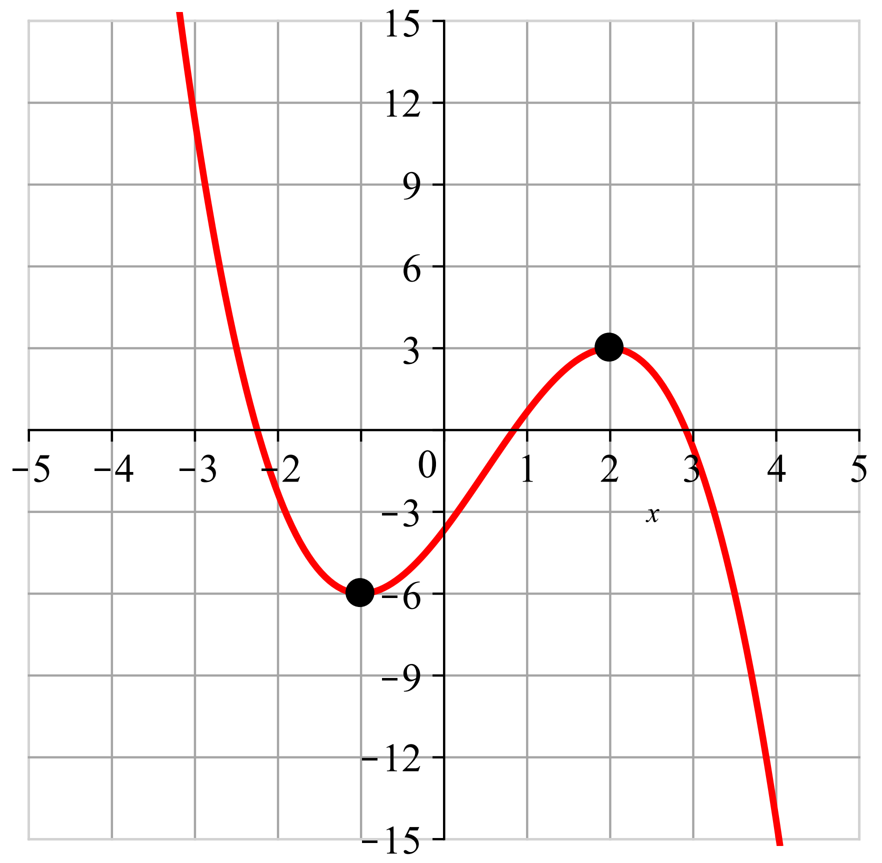

Example 3. $f(x) = -\pi + \sin x + \pi \cos x - x \cos x$ **for** $-3\pi \leq x \leq 3\pi$

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x	-3π		-2π		$-\pi$		0		π		2π		3π
$f'(x)$	0	$+$	0	$-$	0	$+$	0	$-$	0	$-$	0	$+$	0
$f(x)$	-5π	$\nearrow$	2π	$\searrow$	-3π	$\nearrow$	0	$\searrow$	$-\pi$	$\searrow$	-2π	$\nearrow$	π

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$f'(x)$	0	+	0	−	0	+	0	−	0	−	0	+	0
$f(x)$	-5π	↗	2π	↘	-3π	↗	0	↘	$-\pi$	↘	-2π	↗	π

$f(x)$
increases on $[-3\pi, -2\pi]$,
decreases on $[-2\pi, -\pi]$,
increases on $[-\pi, 0]$,
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$f(x)$	-5π	$\nearrow$	2π	$\searrow$	-3π	$\nearrow$	0	$\searrow$	$-\pi$	$\searrow$	-2π	$\nearrow$	π

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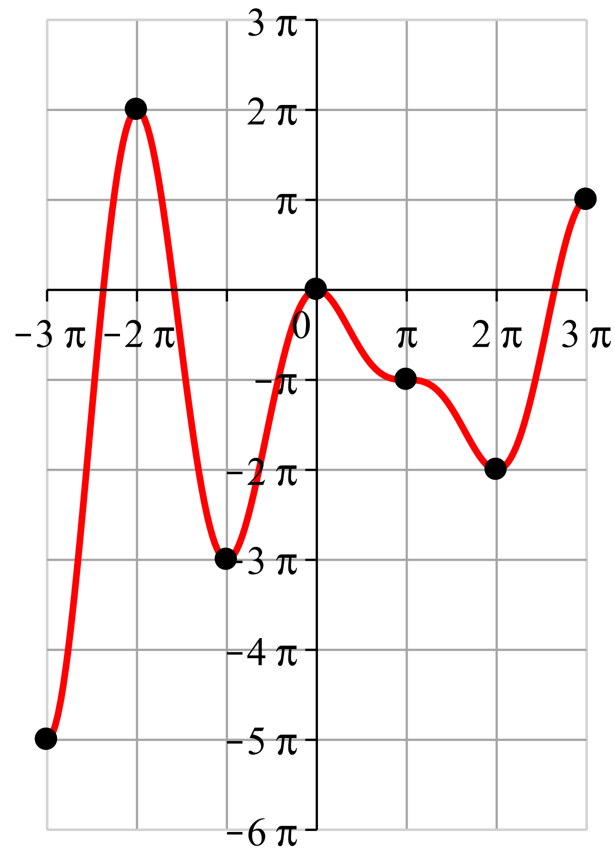

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$$f'(x) = \frac{2(x+1)(-4-x)}{(x^2-4)^2}.$$

Critical points:
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x	$-\infty$		-4		$-2^-, -2, -2^+$		-1		$2^-, 2, 2^+$		∞
$f'(x)$	0	$-$	0	$+$	∞,dne,∞	$+$	0	$-$	$-\infty$,dne,$-\infty$	$-$	0
$f(x)$	1	$\searrow$	$3/4$	$\nearrow$	∞,dne,$-\infty$	$\nearrow$	0	$\searrow$	$-\infty$,dne,∞	$\searrow$	1

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$f'(x)$	0	$-$	0	$+$	∞,dne,∞	$+$	0	$-$	$-\infty$,dne,$-\infty$	$-$	0
$f(x)$	1	$\searrow$	$3/4$	$\nearrow$	∞,dne,$-\infty$	$\nearrow$	0	$\searrow$	$-\infty$,dne,∞	$\searrow$	1

$f(x)$ decreases on $(-\infty, -4]$,
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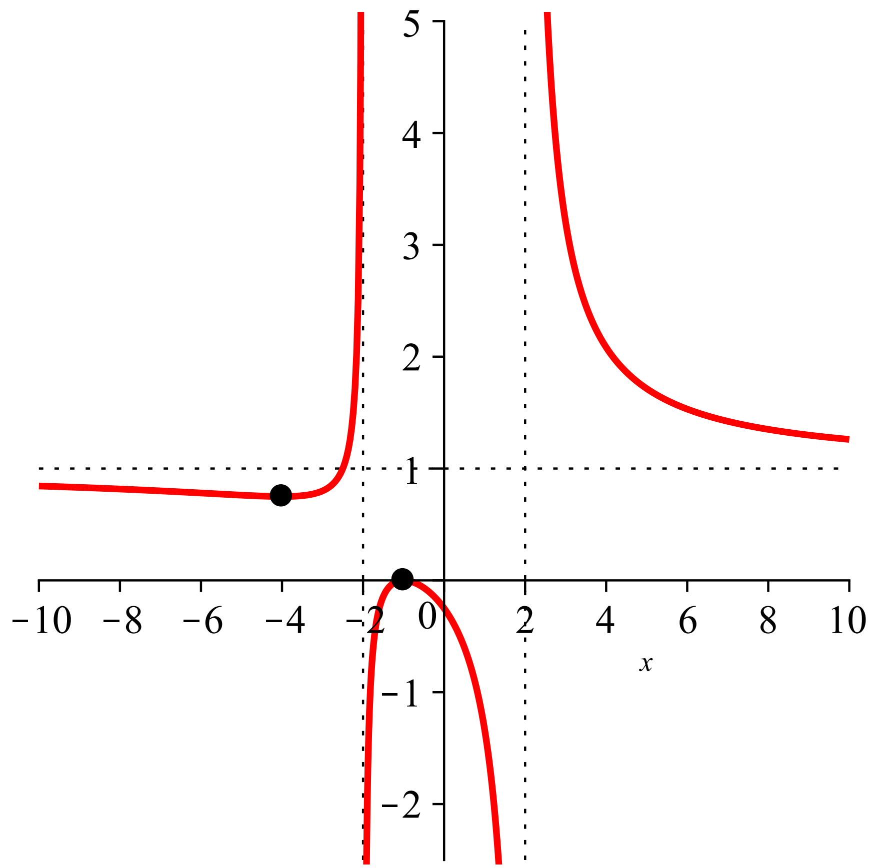

Example 5. $f(x) = \begin{cases} x^3 - 12x & x < 1, \\ -x^2 + 6x - 3 & x \geq 1. \end{cases}$

(a) Find all local extrema of f on $(-\infty, \infty)$.

(b) Find the abs max & abs min of f on $[-3, 4]$.

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$x = -2$, $x = 1$, and $x = 3$.

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x	$-\infty$		-2		1		3		∞
$f'(x)$	∞	$+$	0	$-$	dne	$+$	0	$-$	$-\infty$
$f(x)$	$-\infty$	$\nearrow$	16	$\searrow$	2	$\nearrow$	6	$\searrow$	$-\infty$

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Solution to (a):

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$f'(x)$	∞	$+$	0	$-$	dne	$+$	0	$-$	$-\infty$
$f(x)$	$-\infty$	$\nearrow$	16	$\searrow$	2	$\nearrow$	6	$\searrow$	$-\infty$

Solution to (a):

Loc max: $x = -2,\ 3$.

Loc min: $x = 1$.

Solution to (b): On $[-3, 4]$:

End values: $f(-3) = 9,\ f(4) = 5$.

Critical values: $f(-2) = 16,\ f(1) = 2,\ f(3) = 6$.

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Critical points:
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x	$-\infty$		-2		1		3		∞
$f'(x)$	∞	$+$	0	$-$	dne	$+$	0	$-$	$-\infty$
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$f'(x)$	∞	$+$	0	$-$	dne	$+$	0	$-$	$-\infty$
$f(x)$	$-\infty$	$\nearrow$	16	$\searrow$	2	$\nearrow$	6	$\searrow$	$-\infty$

Solution to (a):

Loc max: $x = -2, \ 3$.

Loc min: $x = 1$.

Wait! $f(0) = 0$

This is $< f(1) = 2$.

Solution to (b): On $[-3, 4]$:

End values: $f(-3) = 9, \ f(4) = 5$.

Critical values: $f(-2) = 16, \ f(1) = 2, \ f(3) = 6$.

Abs max: $f(-2) = 16$.

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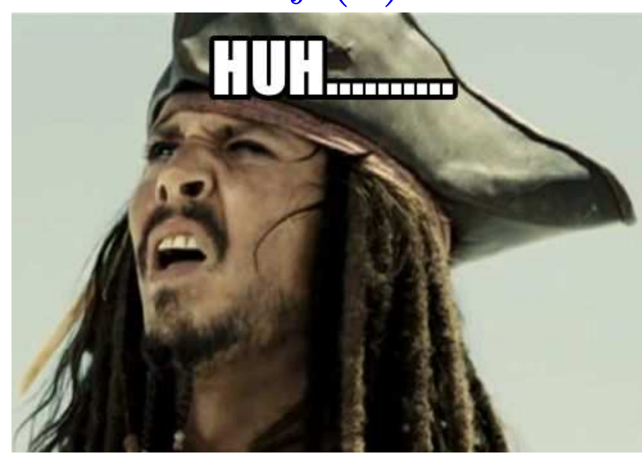

Solution to (b): On $[-3, 4]$:

End values: $f(-3) = 9, \; f(4) = 5$.

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Abs max: $f(-2) = 16$.

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x	$-\infty$		-2		1		3		∞
$f'(x)$	∞	$+$	0	$-$	dne	$+$	0	$-$	$-\infty$
$f(x)$	$-\infty$	$\nearrow$	16	$\searrow$	2	$\nearrow$	6	$\searrow$	$-\infty$

Solution to (a):

Loc max: $x = -2,\ 3$.

~~Loc min: $x = 1$~~. **WRONG!**

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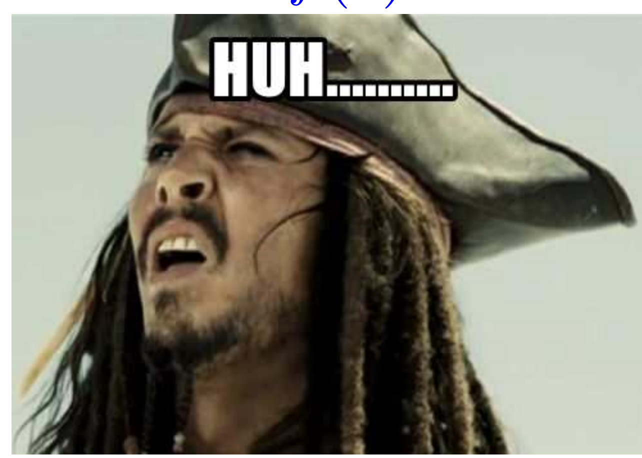

Solution to (b): On $[-3, 4]$:

End values: $f(-3) = 9$, $f(4) = 5$.

Critical values: $f(-2) = 16$, $f(1) = 2$, $f(3) = 6$.

Abs max: $f(-2) = 16$.

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Wait! $f(0) = 0$

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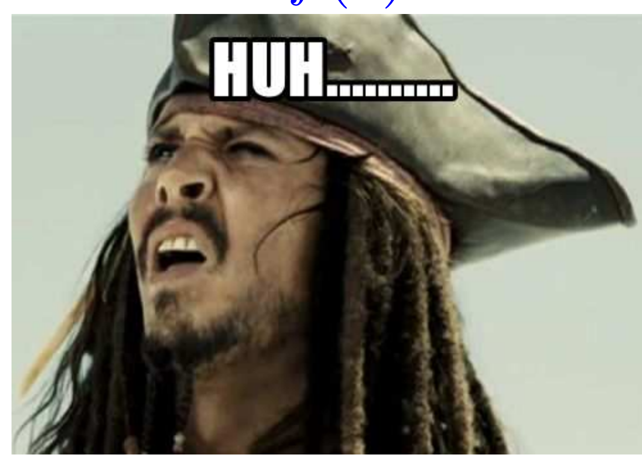

Beware of discontinuities!

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Critical points:
$x = -2$, $x = 1$, and $x = 3$.

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x	$-\infty$		-2		1^-,1,1^+		3		∞
$f'(x)$	∞	+	0	−	,dne,	+	0	−	$-\infty$
$f(x)$	$-\infty$	↗	16	↘	−11,2,2	↗	6	↘	$-\infty$

Evaluate the one-sided limits of f at discont points.

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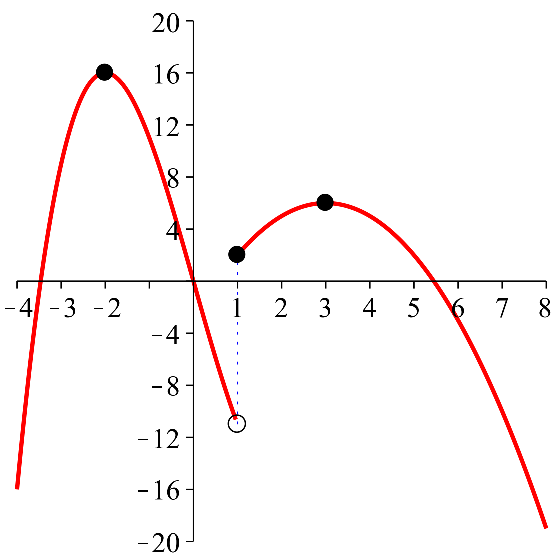

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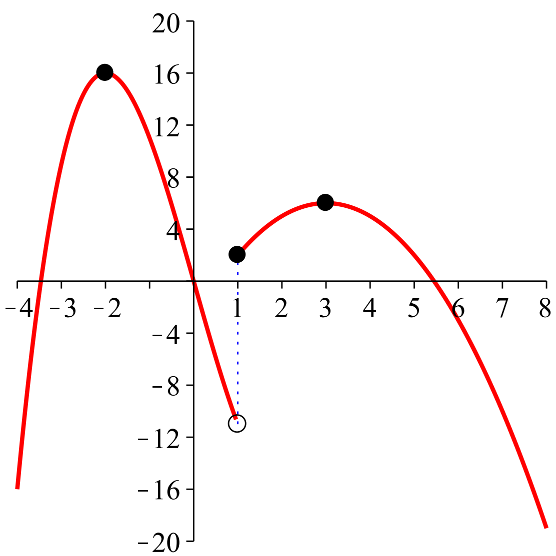

Answer to (a):

Local max: $x = -2,\ 3$.

Local min: None!

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x	$-\infty$		-2		1^-,1,1^+		3		∞
$f'(x)$	∞	$+$	0	$-$	,dne,	$+$	0	$-$	$-\infty$
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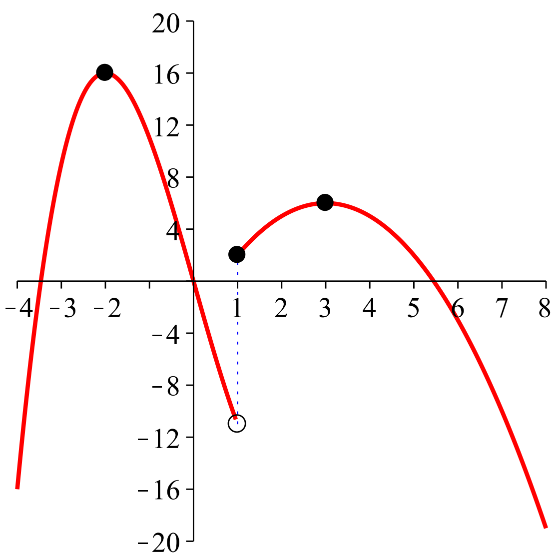

Answer to (a):

Local max: $x = -2,\ 3.$

Local min: None!

Remark: $x = 1$ is a critical point, but is not a local extremum.

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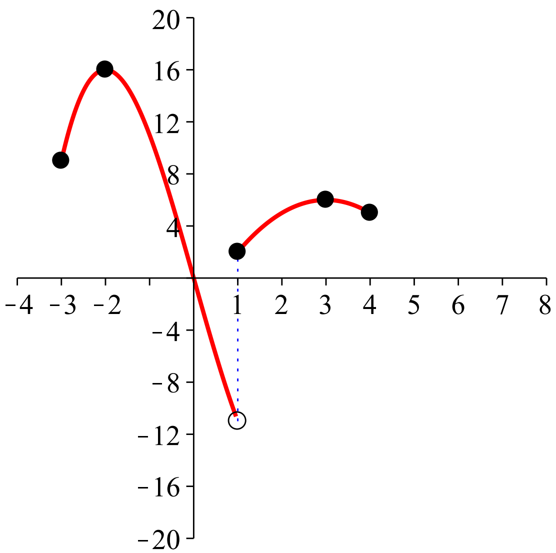
Critical points:
$x = -2$, $x = 1$, and $x = 3$.

x	-3		-2		1^-,1,1^+		3		4
$f'(x)$		+	0	−	,dne,	+	0	−	
$f(x)$	9	↗	16	↘	−11,2,2	↗	6	↘	5



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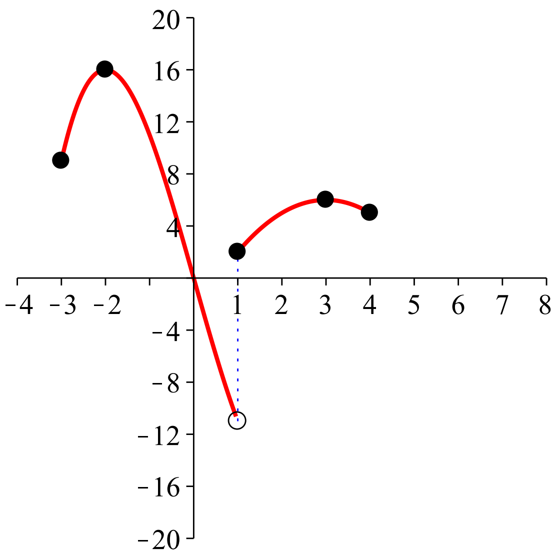
x	-3		-2		$1^-, 1, 1^+$		3		4
$f'(x)$		$+$	0	$-$	,dne,	$+$	0	$-$	
$f(x)$	9	$\nearrow$	16	$\searrow$	$-11, 2, 2$	$\nearrow$	6	$\searrow$	5



Answer to (b):
f on $[-3, 4]$ has abs max at $x = -2$,
but has no abs min.

Example 6. $f(x) = \begin{cases} x^3 - 12x & x \leq 1, \\ -x - 10 & x > 1. \end{cases}$

$$f'(x) = \begin{cases} 3x^2 - 12 & x < 1, \\ \text{dne} & x = 1, \\ -1 & x > 1. \end{cases}$$

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Critical points:
$x = -2$, and $x = 1$.

x	$-\infty$		-2		$1^-, 1, 1^+$		∞
$f'(x)$	∞	$+$	0	$-$	,dne,	$-$	-1
$f(x)$	$-\infty$	$\nearrow$	16	$\searrow$	$-11, -11, -11$	$\searrow$	$-\infty$

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$f(x)$	$-\infty$	$\nearrow$	16	$\searrow$	$-11, -11, -11$	$\searrow$	$-\infty$

$f(x)$
increases on $(-\infty, -2]$, and
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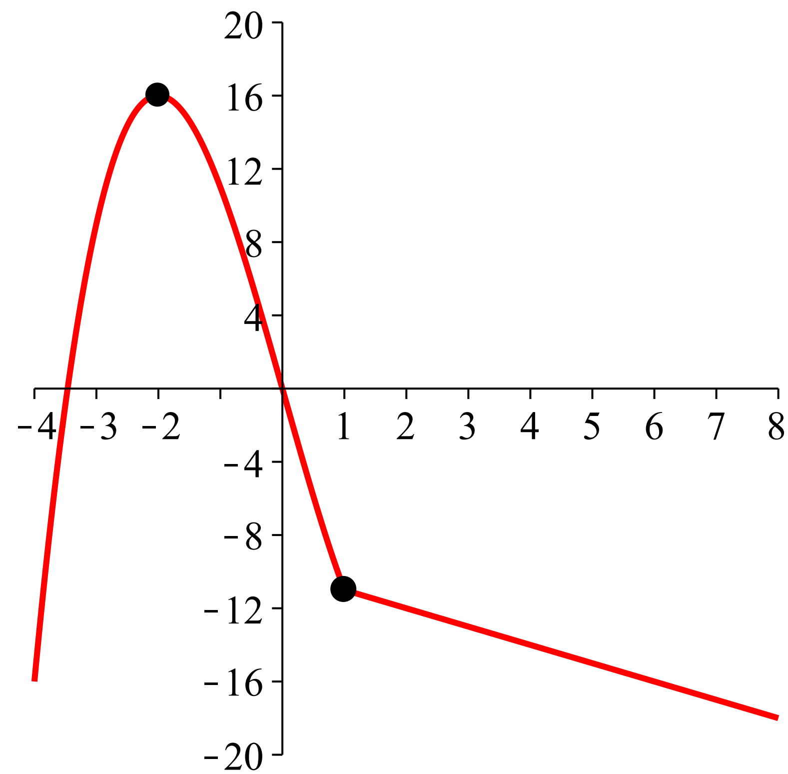

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But $f(1) = -11 < f(2) = 3$.

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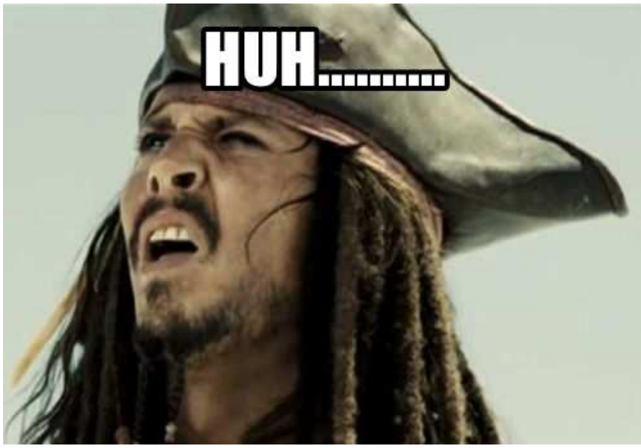

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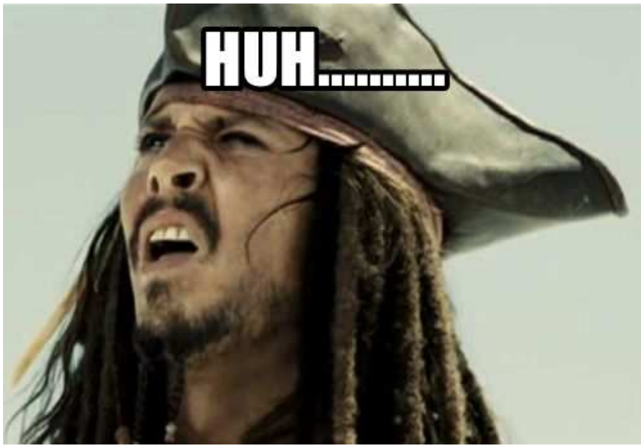

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WRONG!

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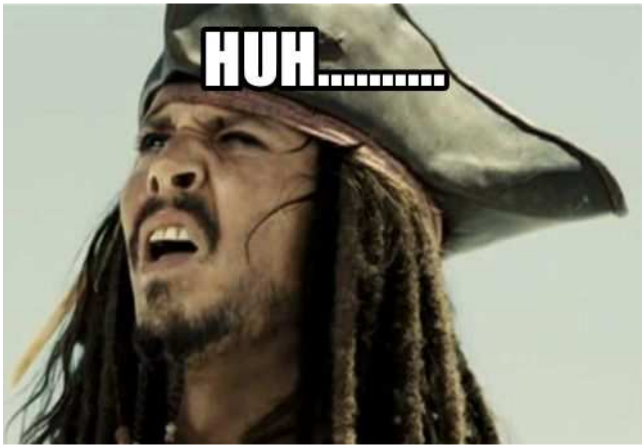
Beware of discontinuities!

But $f(1) = -11 < f(2) = 3$.



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Correct Solution:

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Loc max: $x = -2$.
Loc min: $x = 1$.

Remark: f is not decreasing on $[-2, \infty)$, even though it is decreasing on $[-2, 1]$ and on $(1, \infty)$ separately.

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